

## A NEW METHOD FOR SOLVING QUARTICS

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A new method for solving the general quartic equation is presented, which is different from the methods available in the literature.

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### 1. Introduction

In this paper we describe a new method to solve the general quartic equation. The method is different from the well-known Ferrari's method, or any other earlier method [Wikipedia, Kulkarni (2006)]. In this method the given general quartic equation is compared with an easily decomposable quartic equation, resulting in its decomposition into a product of two quadratic polynomials.

### 2. The proposed method

Consider the general quartic equation in  $x$ , for which solution is sought, as given below.

$$x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0 \quad (1)$$

where the coefficients,  $a_0$ ,  $a_1$ ,  $a_2$ , and  $a_3$ , are real. Consider another quartic equation shown below.

$$(x^2 + b_1x + b_0)^2 - p(x + c_0)^2 = 0 \quad (2)$$

where  $b_0$ ,  $b_1$ ,  $c_0$ , and  $p$  are unknowns to be determined. We attempt to represent the given quartic (1) in the form of (2) which is easily decomposable into two factors as indicated below.

$$[x^2 + (b_1 - p^{1/2})x + b_0 - p^{1/2}c_0][x^2 + (b_1 + p^{1/2})x + b_0 + p^{1/2}c_0] = 0 \quad (3)$$

For this purpose, we expand (2) and rearrange as shown below to facilitate comparison with (1).

$$x^4 + 2b_1x^3 + (b_1^2 + 2b_0 - p)x^2 + 2(b_0b_1 - pc_0)x + b_0^2 - pc_0^2 = 0 \quad (4)$$

Equating the corresponding coefficients of quartic equations (1) and (4), we obtain four equations in four unknowns,  $b_0$ ,  $b_1$ ,  $c_0$ , and  $p$ , as shown below.

$$2b_1 = a_3 \quad (5)$$

$$b_1^2 + 2b_0 - p = a_2 \quad (6)$$

$$2(b_0b_1 - pc_0) = a_1 \quad (7)$$

$$b_0^2 - pc_0^2 = a_0 \quad (8)$$

Using the above four equations the unknowns,  $b_0$ ,  $b_1$ ,  $c_0$ , and  $p$ , are determined. From (5)  $b_1$  is evaluated as,  $b_1 = a_3/2$ , and from (5) and (6)  $b_0$  is expressed as shown below.

$$b_0 = [p + a_2 - (a_3^2/4)]/2 \quad (9)$$

Using (9) we eliminate  $b_0$  from equation (7), to get an expression in  $p$  and  $c_0$ , as shown below.

$$2pc_0 = (a_3/2)[p + a_2 - (a_3^2/4)] - a_1 \quad (10)$$

Again using (9),  $b_0$  is eliminated from (8) resulting in another expression in  $p$  and  $c_0$  as given below.

$$4pc_0^2 = [p + a_2 - (a_3^2/4)]^2 - 4a_0 \quad (11)$$

Squaring equation (10) and then dividing it by (11), we eliminate  $c_0$ , and obtain an expression in  $p$  as shown below.

$$p\{[p + a_2 - (a_3^2/4)]^2 - 4a_0\} = \{(a_3/2)[p + a_2 - (a_3^2/4)] - a_1\}^2 \quad (12)$$

To simplify equation (12) we make following substitutions.

$$\begin{aligned} F_1 &= a_2 - (a_3^2/4) \\ p_1 &= p + F_1 \end{aligned} \quad (13)$$

With these substitutions and after some rearrangement, equation (12) will appear as:

$$p_1^3 - a_2p_1^2 + (a_1a_2 - 4a_0)p_1 + 4a_0F_1 - a_1^2 = 0 \quad (14)$$

Notice that (14) is a cubic equation in  $p_1$  with real coefficients, and when solved over the real field it provides at least one real root. Once  $p_1$  is known  $p$  is evaluated from (13), and then,  $b_0$  and  $c_0$  are found out from (9) and (10) respectively. Having determined all the unknowns, we are in a position to represent the given quartic equation (1) in the form of (2), whose factors are quadratic polynomials as given in (3). Equating each of these factors to zero, and solving the resulting quadratic equations, we get all the four roots of given quartic (1) as shown below.

$$\begin{aligned} x_1 &= \{ - (b_1 - p^{1/2}) + [(b_1 - p^{1/2})^2 - 4(b_0 - p^{1/2}c_0)]^{1/2} \} / 2 \\ x_2 &= \{ - (b_1 - p^{1/2}) - [(b_1 - p^{1/2})^2 - 4(b_0 - p^{1/2}c_0)]^{1/2} \} / 2 \\ x_3 &= \{ - (b_1 + p^{1/2}) + [(b_1 + p^{1/2})^2 - 4(b_0 + p^{1/2}c_0)]^{1/2} \} / 2 \\ x_4 &= \{ - (b_1 + p^{1/2}) - [(b_1 + p^{1/2})^2 - 4(b_0 + p^{1/2}c_0)]^{1/2} \} / 2 \end{aligned} \quad (15)$$

### 3. Numerical example

Let us solve the following quartic equation using the method proposed here:

$$x^4 - x^3 - 19x^2 - 11x + 30 = 0.$$

For this quartic equation  $b_1$  is found out as:  $b_1 = -1/2$ .  $F_1$  is evaluated as:  $F_1 = -(77/4)$ , and the cubic equation (14) in  $p_1$  is given by:

$$p_1^3 + 19p_1^2 - 109p_1 - 2431 = 0.$$

The above cubic is solved [Kulkarni, 2006], and its roots are obtained as: 11, -13, -17.

With  $p_1 = 11$ , we determine  $p$ ,  $b_0$ , and  $c_0$  as:  $p = 121/4$ ,  $b_0 = 11/2$ , and  $c_0 = 1/11$ . We calculate back the coefficients,  $a_0$ ,  $a_1$ ,  $a_2$  and  $a_3$ , and obtain the values as: 30, -11, -19, and -1, respectively. Thus we ensure that,  $p_1 = 11$ , is the desired value required to be used to find the roots of quartic in this example. Using the expressions given in (15), the roots of quartic,  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$ , are determined as: 5, 1, -2, and, -3 respectively.

### 4. Conclusions

The general quartic equation is solved using a new method, which compares the given quartic with an easily decomposable quartic. A numerical example is solved using the proposed method.

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### **References**

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