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A PATTERN MODEL FOR ASSESSING WORK COMPETENCIES USING PETRI NETS

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This paper presents a formal framework for human behavior modeling, analysis and validation. It proposes a competencies model for representing the position profile of an employee/candidate and it uses the Hierarchical Decision Process Petri nets (HDPPN) for simulating the behavior of an employee/candidate in a given business process. The competencies model consists of a definition model, an evaluation process model, and a graphic representation model. The most critical activity in the development and the design of a position profile is that of a business process model. Business process modeling is based on a business strategy transformation, where high-level business strategies are refined up to the point that they can be described in terms of the activities (needed to achieve a certain tactical business strategy given only in terms of goals and strategies). Every position profile is designed to take into account its corresponding business processes. In this sense, each activity of a given process is associated with a competency. Consequently, each competency is transitively aligned with the business strategy. The competencies model provides a tool to define an "employee prototype" (ideal or expected employee) in terms of competencies. The evaluation process is implemented by partition functions which are defined by features that are mostly qualitative: classes, competencies, sub-competencies. For evaluating an “employee prototype” we establish metrics to rate the domain degree (the minimum expected value) needed to satisfy the sub-competencies of the profile. Metrics consist of a measurement scale and a measurement method. They are preferably measured qualitatively by using certainty linguistic values, without discarding quantitative measurement. For assessing an employee/candidate, we use psychological tools (tests, interviews, assessment center, etc.) to rate the sub-competencies of the partition functions. As a result of the evaluation process, the model produces a "closeness degree" of how close the applicant is to what is expected from an “ideal” employee. To represent the assessments, we proposed a special pie chart, where each slice (or partition) represents a competency. The HPPN is used to validate the performance of an applicant imitating the possible behavior of an employee/candidate in the business process. As a result, the HDPPN generate the utility value associated to the simulated behavior of the “employee prototype.” It considers the value of the competencies determined in the position profile, producing a weighted utility value for each competency. As a final point, it is obtained approach of how close the behavior is of the applicant to what is expected from an “employee prototype" in the business process. A software tool implements the model of the competencies. For illustration purposes an application example is given.

Keywords: Competencies, Human Behavior, Business Process, Petri nets, Decision Process, Lyapunov Theory, Information Technology Strategic Planning, Strategies and Goals, Software.

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1. Introduction

1.1. *Brief survey*

A company's success depends on the ability to evolve with the market, not to just respond to it. In response to the competitive pressures enforced by the customer demands and the constant changes on the conditions of the environment, many companies are re-thinking the way they do business [Hammer (1990)]. The environmental turbulence has created a need for dynamic business processes, and companies are looking for models that can evolve and adapt efficient business processes, human behavior, and competencies aligned to the changing conditions and the changing business strategies.

As a consequence, there has been an increasing demand for human behavior modeling and competency models that respond to business process dynamics and deal with the domains of application beyond the traditional approaches ([Outerbridge (1979)], [Salgado, Viswesvaran and Ones (2002)], [Schmidt and Hunter (1998)], [Steel, Huffcutt and Kammeyer-Mueller (2006)], [Tenopyr (1996)]). Effectiveness of such a model depends largely on the ability to represent the domain of the problem in such a way as to permit natural and rigorous descriptions within a methodological framework.

The concept of competence, in the sense of being used in psychology and human resources, was created by David C. McClelland [McClelland (1973)] in his work entitled "Testing for competence rather than for intelligence." Spencer [Spencer and Spencer (1993)] refers to a competence as "an underlying characteristic of an individual that is causally related to a criterion reference group's effective performance in a job or situation."

Despite the fact that the concept of competence has been widely recognized in the business and academic literature ([McClelland (1973)], [McLagan (1997)], [Parry (1998)], [Spencer and Spencer (1993)]), there is little consensus on the meaning of competence. Buford and Lindner [Buford and Lindner (2002)] define competencies as a group of related knowledge, skills, and abilities that affect a major part of an activity.

We conceptualize the terms “skills, knowledge, and abilities” as different but related concepts [Rothwell and Lindholm (1999)]. Skills are observable competencies needed to perform a learned psychomotor act. Knowledge is information applied directly to the performance of a given activity. Abilities are competencies needed to perform an observable behavior or a behavior that results in an observable product.

The strategic modeling of position competencies for creating a strategic and competitive advantage is a critical factor for every organization ([Athey and Orth (1999)], [Dalton (1997)], [Hoffmann (1999)], [Kochanski (1997)], [Pappas, Flaherty and Hunt (2007)], [Spencer and Spencer (1993)]). The most critical activity in addressing this concern is to provide a closer alignment and a continuous adaptation of the competencies to the business strategy and the organizational model as a whole.

Related researches on the competencies-modeling mechanism are reported in the literature from the 1990s to date ([Chong et al. (2000)], [Currie and Darby (1995)], [Ellstrom (1997)], [Galinec and Vidović (2006)], [Hoffmann (1999)], [Robotham and Jubb (1996)], [Schmidt and Hunter (1998)]). Few of these methods fail to support the necessary competencies constraints, and the others establish a restricted linkage between business strategy, organizational model, and competencies requirements. Therefore, this problem is still a challenging issue ([Chong et al. (2000)], [Dalton (1997)]).

Organizations use competencies models in different ways [Yeung, Woolcock and Sullivan (1996)]. For instance: assessment instrument, recruitment and selection, career development, coaching, counseling, mentoring, training, and as a behavioral requirement benchmarking tool.

The main problem in human resources is that the terms "selecting" and "job performance" are in practice ambiguously defined. The confusion may be attributed to several reasons, as for example:

1. Position competencies are not defined by a single idea, but rather a multidimensional concept. The definition of dimensions depends on a business process model.
2. For any competence there are different levels of abstraction. When people talk about selecting and job performance, they could refer to it in its broadest sense, whereas some others might refer to it by its specific meaning.
3. Selecting people for a specific position is not determined by a single perception, but by a sum of perceptions (Confucius: the sum of perceptions is equal to reality).

Selecting people is part of our everyday language but the techniques are confusing.

A common perspective of the evaluation of people is that it is imprecise. It can be discussed and judged, but cannot be weighted or measured. Therefore, to many people, selecting is based on an intuitive process supported by intuition and personal perception. Terms such as "I like him", "I think that he is good" and similar concepts show how people reason about something uncertain and diffuse. This perception shows the fact that people perceive, understand, interpret and handle people selection in different ways. The implication of this perception is that selection cannot be controlled and managed, nor can it be quantified. This view is in contrast to the fact that selecting can and should be defined, measured, and managed ([Bouhuys et al. (1996)], [Ilardi et al. (1996)], [Schmitt and Chan (1998)]).

Selecting an ideal or expected employee is crucial for organizational success. Selection methods that allow firms to identify the right people (from a pool of applicants) are vital components. The selection methods used depends on the job position [Fisher, Schoenfeldt, Shaw (2003)]. They include review of resume/curriculum-vitae, interviewing, testing, assessment centers, etc. For instance, tests are used to measure abilities like: knowledge, aptitude, intelligence, personality, integrity, interest, etc. [French (2003)]. Tests appear to provide an objective evaluation that can be validated. Assessment centers are most often used for promotion to managerial positions. They allow applicants to try on senior roles in a simulated environment.

1.2. *Main questions*

Most firms use more than one selection method to collect information about applicants. This work is related with how to integrate (sum) the evaluation of these methods (perceptions) and obtain a result (reality).

There are at least three important issues encountered in modeling and quantifying a position profile:

- which competencies will be considered and how will they be defined?,
- how this competencies will be rated according to the evaluation process model?, and
- how these competencies will be graphically represented?

1.3. *Main results*

We propose a pattern model for assessing work competencies using Hierarchical Decision Process Petri Nets [Clempner (2010)].

The most critical activity in the development and the design of a position is that of a business process model. The method is based in business strategy decomposition. High-level business strategies are refined up to the point when they reach a tactical business strategy level, described only in terms of goals and strategies. The importance of being able to clearly link the business processes with the business strategy is highlighted by the concept of business reengineering [Hammer (1990)]. The notion of business strategy decomposition is adopted to represent the process of business-strategy refinement. Activities are considered as operationalizations of goals and are applied in accordance with the strategies needed to achieve these goals. Thus, the decomposition process results in a set of primitive actions such as "order a product". Strategies are expressions that define valid state transitions in the business process. In fact, strategies specify the event occurrences and they represent either integrity rules or control operations. Since the business strategy decomposition determines actions-sequence applications, a process can be ordered introducing a partial-ordered relation. It is important to note that any business process ultimately ends because real processes are finite. The method considers a dynamic application domain, since the organizational model is able to modify its structure and respond appropriately to the changes in the business strategy.

Partially ordered transitions PN are used for business process representation, taking advantage of the well-know properties of this approach namely, formal semantic, graphical display and wide acceptance by practitioners. A HDPPN ([Clempner (2008)], [Clempner (2010)]) model of a business process gives a specific and unambiguous description of the behavior of the process. Its solid mathematical foundation has resulted in different analysis methods and tools. Despite of the formal background, Petri net models are easy to understand.

Each activity in the business process is associated with a competency. The evaluation process model provides a tool for constructing partition functions to define an ideal "employee prototype" in terms of competencies. The partition functions construction is based on competencies refinement (note that these kinds of structures are widely used in the software product evaluation area [ISO/IEC 9126 (1991)]). Sub-competencies are identified from the business process model; note that the sub-competencies identification depends on the application domain. However, each sub-competency is grouped into competencies. For clarity, the competencies are arranged in competency classes.

On the one hand, for evaluating an ideal "employee prototype" metrics are defined to rate the "domain degree" (the minimum expected value) needed to satisfy a sub-competencies of the partition functions. On the other hand, for evaluating an employee/candidate psychological tools are used to rate the sub-competencies of the partition functions. As a result, the model produces a "closeness degree" of how close the applicant is to what is expected from an ideal "employee prototype".

To represent the evaluations we proposed a special pie chart, where each slice or partition represents a competency. Pie charts are represented for three different levels: competency class, competency and sub-competency. In this sense, we will consider the terms competency class, competency, and sub-competency equivalent to the term competence.

Our competencies model has a closer alignment with the business strategy. Our approach is an integral part of an information technology strategic planning (ITSP) model

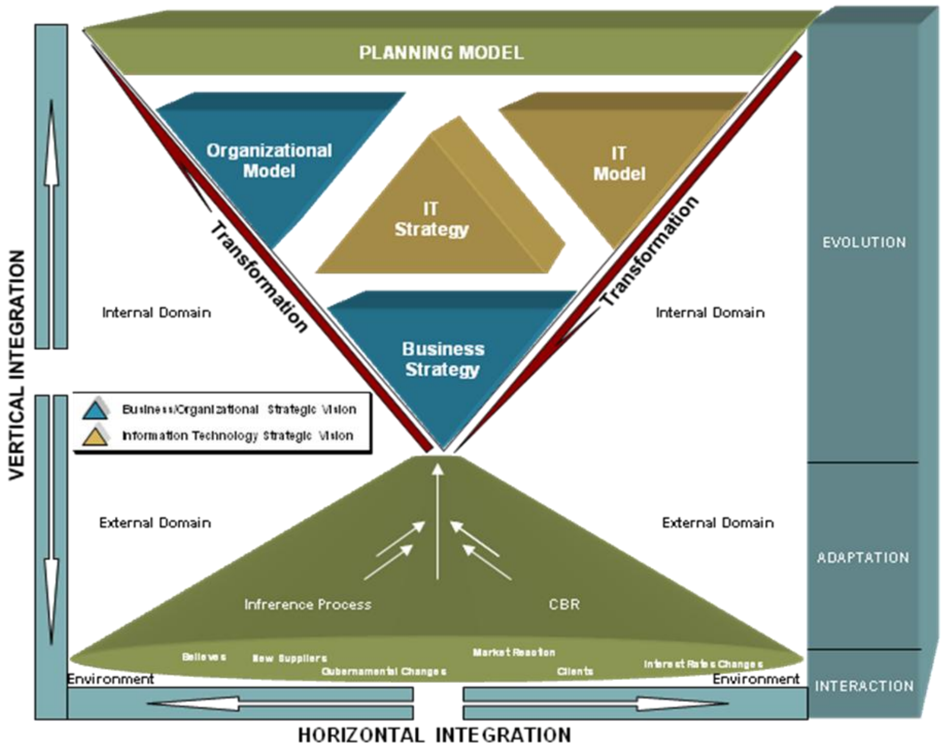

Fig. 1. ITSP Model

and its methodology. The ITSP model considers a dynamic application environment, which integrates the strategic visions of the business/organization and the IT strategic vision in a resulting unified vision. Its conceptualization is based on three fundamental concepts: interaction, adaptation and evolution. The ITSP methodology is organized in fifteen modules. The organization structure module deals with the competencies model.

1.4. *Organization of the paper*

The rest of the paper is structured in the following manner. The next section describes the basic formalism of the ITSP model and its methodology. Then, in section 3 we describe the competencies model in terms of the basic concepts and the graphical notation. Thereafter, we discuss the issues associated to the competence model method. Subsequently, in section 4 we present an application example using HDPPN. The paper concludes presenting the current status of the work, and future research directions are given in the section 5. An appendix presents the necessary mathematical background and terminology needed to understand the paper.

2. ITSP Conceptual Model and Methodology

In the model represented in fig. 1, the real world is composed by entities representing physical things (people, governments, enterprises, etc.). These entities are related in terms of goals, beliefs, etc. Entities under events generation change the environmental conditions. They take particular strategic positions through the network of relationships with other entities, where they play different roles. The model is based on three fundamental concepts: interaction, adaptation and evolution.

The interaction concept represents the dynamic behavior of the environment, leading to the incorporation or rejection of beliefs and facts related with environment conditions. Interactions are established by the relationships between the roles that each entity plays in the domain of application. The behavior of the environment is induced by the interaction of the entities.

When an incident happens (beliefs, market reactions, etc.), and it changes the environment conditions, it is called an event. Each entity has the option to consider an event occurrence and it incorporates or rejects the facts related to changes in the environment. The acceptance or rejection will depend on the entities interest. Some examples of conditions that can be accepted are: economic plans changes, political beliefs, new technological tendencies, interest rate growth, etc.

The adaptation includes business strategies using a logic inference method, which uses beliefs and facts in order to generate new business strategies. This is a dynamic process where old business strategies are replaced by those corresponding with the present environmental state. In the real world, there are always assumptions which, if proven to be unfounded, can be easily corrected. The environmental changes always take place in the curse of events that invalidate previous states. On the other hand, non-monotonic reasoning shows an opposite fact to this problem. It simply allows the retraction of 'truth' whenever contradictions arise by forcing the incorporation of new beliefs.

Evolution is a process in which the business strategy is transformed into operative and IT components (the organizational model, the human resources, the IT model and the planning model). It considers a dynamic application domain which integrates the business/organizational strategic visions and the IT strategic vision in a resulting unified vision.

The evolution process is represented by an inverse pyramid where business strategy represents the "axioms" of the archetype of the organizations. These axioms are considered as true fundamental principles, in virtue of the fact that they are congruent with the reality of the environment. In every case, the ITSP tries to be in contact with the real world in order for its construction to be logically coherent. The organization propositions [Henderson and Venkatraman (1993)] (the organization model, the human resources model, the IT model -- IT strategy [King (1978)] -- and the planning model) are deduced from the axioms through a logical inference method. Thus, every proposition is true if it can be deduced from the axioms.

This definition is in agreement with the fact that the efficiency of an enterprise and the effective use of the IT depend on the concordance that exists with the business strategy. If the business strategy is incompatible with the physical structure of the enterprise and the configuration of the IT, then the functionality of the organizational areas will be inefficient. It is important to note that the organizational axioms are not necessarily absolute, but they evolve in accordance with the internal and external changes of the

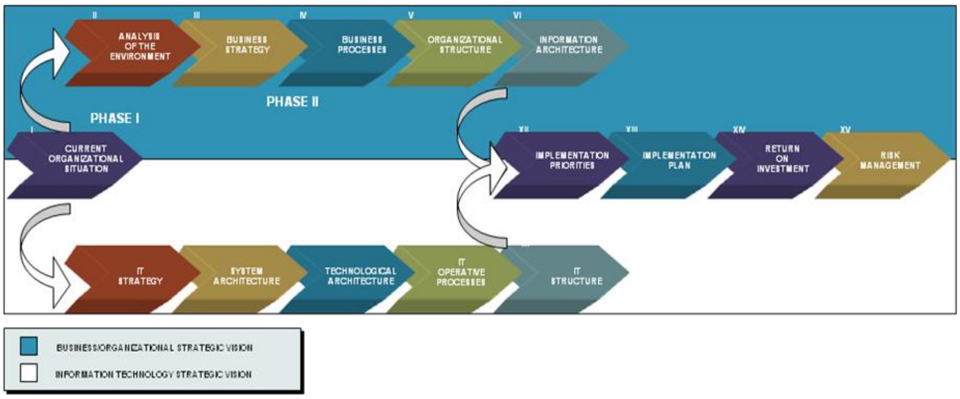

Fig. 2. ITSP Methodology

environment (changes in an organization are limited, i.e. an enterprise that sells computational equipment can be transformed into an enterprise that sells telecommunication equipment, but would be very difficult to transform it into a gas station).

The ITSP methodology (Fig. 2) is organized in fifteen modules which are divided in four phases, and conceived in two visions. In addition, it is concerned with creating a business/organizational vision, which provides the critical information inputs, and it also forms the foundations for later stages of planning. It creates as well a vision of the IT, which exploits new technological solutions and it improves the enterprise situation. The human resources structure module deals with the competencies model. This paradigm is in concordance with the ITSP conceptual model.

3. Conceptual Model of Competencies

3.1. *Model of Competencies*

In the model, there are three essential subjects to understand how to represent and assess a position profile: 1) the competencies definition model, 2) the evaluation process model, and 3) the graphic representation model.

The first subject is concerned with the competencies that should be defined for a position profile. Every position profile is particularly outlined under the support of a business-process model. That is, the establishment of the competencies of a position profile depends on the activities that an employee performs in a business process. The business process model determines the scope and how competencies are hypothesized to explain or predict most of the predictable variance in individual performance within this domain. For example, if the job domain were defined as "managerial" the following question arises: are the competencies which represent the cognitive and the psychometric abilities sufficient to describe most of the criterion variance? or would the inclusion of the competencies related with emotional intelligence ([Goleman (1996)], [Merten (2003)]) enhance the prediction? Cognitive, psychometric, and emotional intelligence

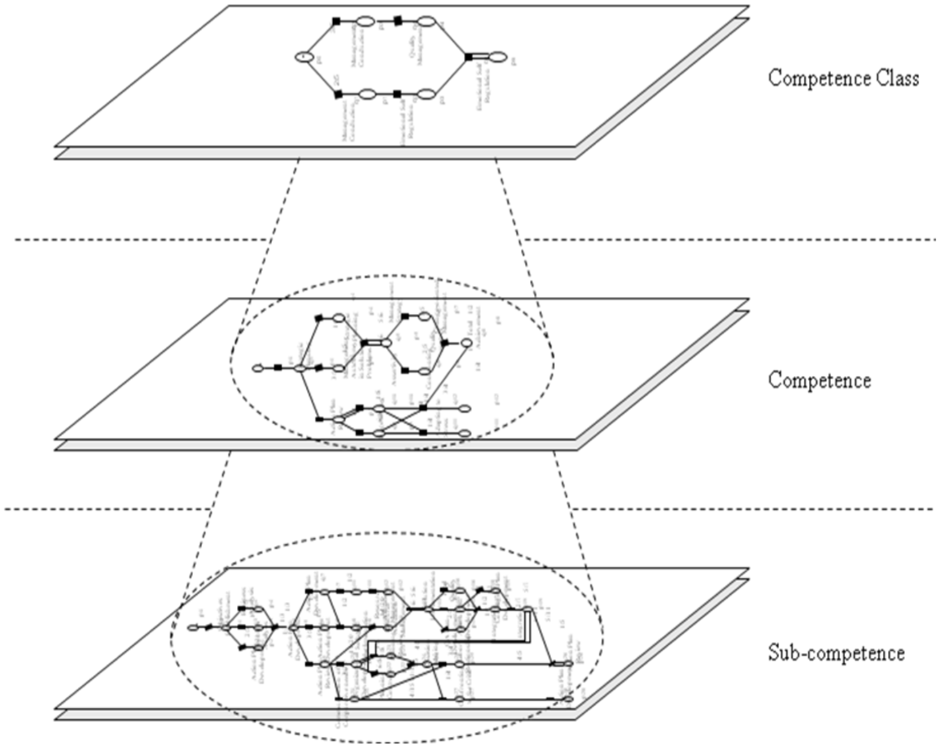

Fig. 3. HDPPN Competencies level

competencies could be included, but it would be with a relative importance (some studies suggest that the relative importance of the emotional intelligence in managers must be between 45 to 65 percent).

In early practices, many competencies have been proposed and used for the evaluation of job positions, but the specific competencies set selected depended on the opinion and the point of view of the designer. They were also based on the "experience." Therefore, it was suggested that a limited number of competencies for cognitive and practical reasons be used. However, the competencies definitions cannot be discussed in vacuum and there must be a reference to the business process model.

The business-process-model method is based on the decomposition of the business strategy. High-level business strategies are refined up to the point where they reach a tactical business strategy level, described only in terms of goals and strategies. The notion of the decomposition of the business strategy is adopted to represent the refinement process of the business strategy. These types of activities are considered as goals "operations" and they are applied in accordance with the strategies needed to achieve these goals. Thus, the process of the decomposition results in a set of primitive actions such as "ordering a product." The strategies are expressions that define valid state transitions in the process of a business. In fact, these strategies specify how an event occurs, and they represent either integrity rules or control operations. Since the decomposition of the business strategy determines actions-sequence applications, a process can be ordered by introducing a partial ordered relation (Fig. 3). The method considers a dynamic application of the domain since the organizational model is able to modify its structure and respond appropriately to the changes in the business strategy.

HDPPNs (Fig. 3) are used for the representation of the business process, taking advantage of the well-known properties of this namely approach, the formal semantic, the graphical display, and the wide acceptance by practitioners ([Clempner (2008)], [Clempner (2010)]). The HDPPN model of a business process gives a specific and unambiguous description of the behavior of the process. Its solid mathematical foundation has resulted in different analysis methods and tools.

The second subject is how the competencies will be measured. When a competency is actually defined, it is classified as well. A competency class is defined as a collection of competencies which have common properties. Competency classes depend on the application domain. The competency classes play a fundamental role in the model of a position profile, because each competency class determines the first level of relative importance. However, competencies are complex concepts, and their properties are insufficient to define a competency in detail. It is necessary that each competency be refined into sub-competencies structured in a hierarchical way. The model supports an n-level decomposition competencies, though for a practical purpose, we just used three levels: competency class, competency and sub-competency. For the purpose of this example, we will confuse and understand the terms "competency class, competency and sub-competency" as competencies.

For evaluating an "employee prototype" we establish metrics to rate the "domain degree" (the minimum expected value) needed to satisfy the sub-competencies of the partition functions [Clempner and Tornes (2004)]. They consist of a measurement scale and a measurement method. One or more metrics could be selected and defined to evaluate each sub-competency. Metrics are preferably measured qualitatively by using certainty-linguistic values, without discarding quantitative measurement.

Linguistic-certainty values constitute the verbal scale that experts commonly use to express their degree of certainty in the factors of the evaluation. Studies in psychology have shown the practicability of such verbal scales. It is known that people give numerical estimations on a common day situation error, and most of the time they are inconsistent in their judgment precision. However, judgments embodied in linguistic descriptors appear consistent in this same situation. Each linguistic value is represented by a fuzzy interval, i.e., the function membership of a fuzzy set on the real line is shown in the space represented by [0,1].

For evaluating applicants psychological tools are used to measure sub-competencies of the partition functions. Examples of psychological tools are: psychometric tests, emotional intelligence tests, feed-back 360º, assessment centers, interviews, and some others.

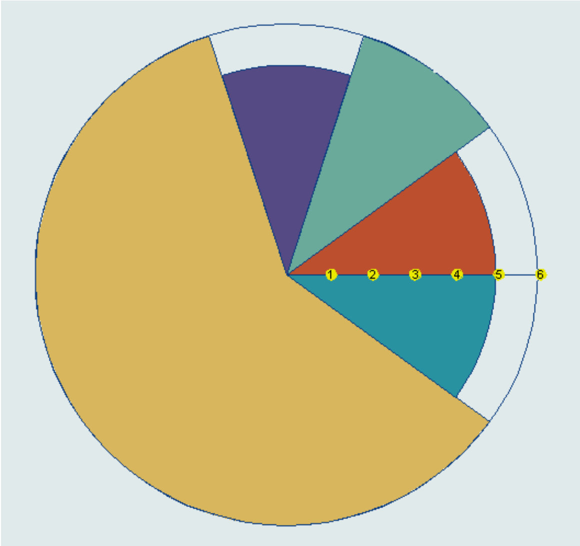

Fig. 4. Competencies Graphic Representation Model

The third subject is how the evaluation of the competencies will be represented. The graphic representation model plays a fundamental role in the interpretation of the evaluation. The position profile should be expressed as simple as possible, considering that the properties of the position profile require different rating levels and weights. In this sense, we suggest a special pie chart to represent the model (Fig. 4). In the pie, each slice represents a competency (competency class, competency and sub-competency), where the weight (relative importance) is represented by the angle of the slice and the rating value by its radius. The radius of the angle is scaled in six grades that correspond to the six grade rating levels. The competencies model provides a tool in order to define an "employee prototype" (the ideal or expected employee) in terms of competencies, and it is also a tool which determines how to evaluate such prototype. The ideal employee is a direct consequence of the ideal business-process model.

The HPPN [Clempner (2010)] is used to validate the performance of an applicant imitating the possible behavior of an employee/candidate in a business process. As a result, the HDPPN generate the utility value associated to the simulated behavior of the applicant. For calculating the utility, the validation process considers the value of the competencies determined in the position profile, producing a weighted utility value for each competence. As a final point, it is an obtained approach of how close the behavior of the applicant is to what is expected from an ideal “employee prototype" in the business process model.

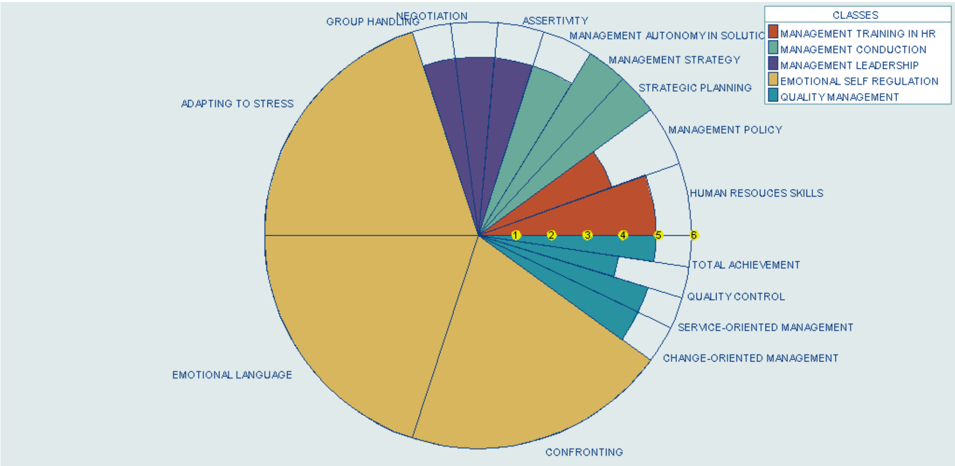

Fig. 5. Competency-Classes Prototype

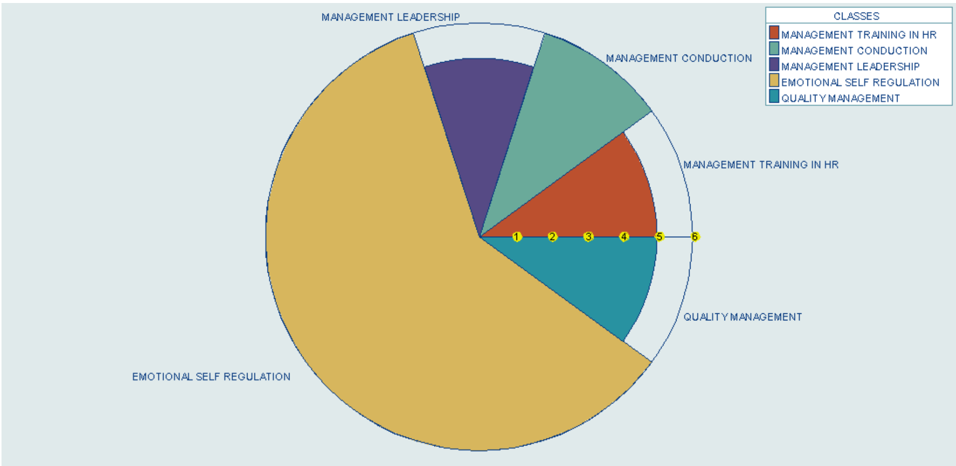

Fig. 6. Competencies Prototype

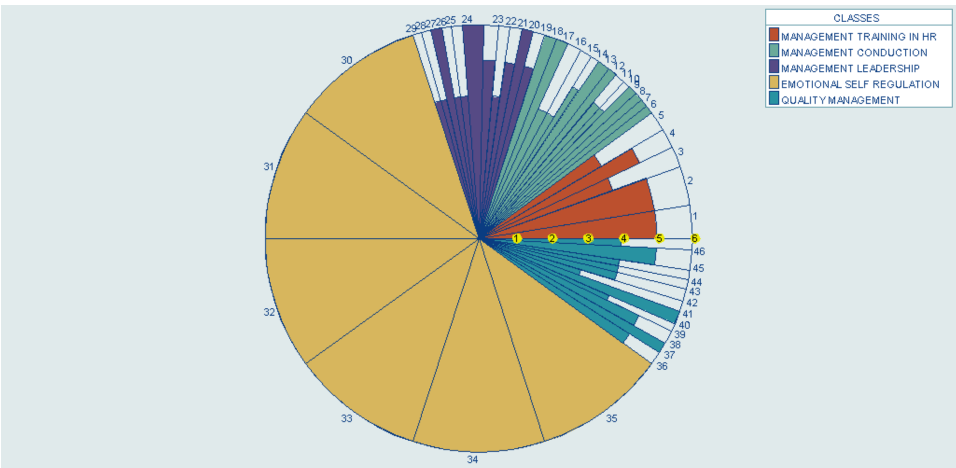

Fig. 7. Sub-competencies Prototype

3.2. *Identification and Validation of Competencies*

The main process for the evaluation of a position profile takes the following steps: 1) design of the business process, 2) definition of the competencies and the profile of the employee/candidate, 3 selection of the metrics and the assessment of the prototype, 4) establishment of the measurement tools and the rating applicants, 5) assessment of the appraisal and, 6) evaluation of the validation process.

(1) Business process model design. - The business process is regarded as a set of activities. These activities are considered as goals "operations," and they are applied in accordance with the strategies to achieve the goals. The strategies determine the legal sequential movements that can be made from any activity to another. The structure of each node in the decomposition of a business strategy is a complex subject, which is defined by the ordered-pair goal strategy. Business processes are modeled using HDPPNs as follows:
   (i) The HDPPN development is carried out via an incremental building by iteratively refining the net of activities (Fig. 3). The development of a HDPPN at the highest level starts with a set of (usually) incomplete and informal net of activities.
   (ii) Places in the HDPPN are inscribed by an informal textual description of the states and transitions by a textual description of the action's functionality.
   (iii) A single transition in a HDPPN at the highest level may be refined in several transitions (that preserve the initial behavior) in a new low-level HDPPN in order to specify the respective activity in more detail. Formal rules for admissible, behavior-preserving refinements of nets have been proposed in [Jensen (1992)] and [Lausen (1988)].

(2) Competencies consideration and prototype definition. - The competency definition is based on the business process, and it is also a refining process. For this propose, we will develop HDPPNs in three different refining levels: competency class, competency and sub-competency. The HDPPN of a competency-class level has associated with each activity’s competency class (not necessarily different). Consequently, the HDPPN at a competency level has associated to each activity, a competency, and the HDPPN at a sub-competency level has associated to each activity, a sub-competency. Note that sub-competencies are a refining of competencies and competencies are a refining of competency classes. For the prototype definition, some remarks must be taken into account:
   (i) The definition of a class depends on the domain of the application.
   (ii) The relative importance for each competency class, competency and sub-competency must be determined but differs depending on the position profile.

(3) Metric selection and prototype evaluation. - To measure the "domain degree" of each sub-competency, defined metrics are selected. Once this task is fulfilled, the rating criteria for each metric are defined. Then, the scores for sub-competencies are calculated and represented in a pie chart with its weight (Fig. 7). In the same way, competencies and competency classes are calculated and represented in a pie chart

(Fig. 6 and Fig. 5 respectively). As a result, an employee is created a prototype (supported by an ideal model of a business process).

(4) Measurement tools and rating applicants. - Depending on the application domain, psychological tools are selected to measure the sub-competencies. Applicants are evaluated, and each sub-competency is measured according to the rating criteria. As a result, a pie chart is produced.

(5) Appraisal. - As a result of the evaluation of the sub-competencies, competencies, and classes, a score is obtained, and a pie chart is produced. Then, the model produces a "closeness-degree" pie chart of how close the evaluated applicant is to what is expected from an ideal “employee prototype”.

(6) Validation process.- We will simulate in the HDPPN the possible behavior of each applicant to obtain an approach of how close the behavior of the applicant is to what is expected from an “employee prototype" in the business process

  (i) By construction of the HDPPN, each activity has associated a utility. The utility value obtained for each sub-competency will be weighted by the value achieved for the sub-competency in the appraisal step. The same process will be performed for the computation of the utility of the competencies and the competencies class. It is important to note that the calculation of the utility at the HDPPN competency level depends on the utility calculation accomplished in the HDPPN sub-competency level, as well as the utility at the HDPPN competency-class level depends on the utility calculated at the HDPPN competency level.

  (ii) Finally, the validation process produces a weighted utility value for each sub-competency, competency and competency class at the corresponding HDPPN level, giving an approach of the behavior of the applicant in the business-process model.

3.3. *Calculation Model of Competences*

Formally, let *S* be a non-empty set of personal competencies and let $f : S \to \mathbb{R}$ be a real function, for instance in human resources or psychology area:

1) $\forall s \in S \ : \ f(s)$ is the measure of the emotional self regulation capacity
2) $\forall s \in S \ : \ f(s)$ is the measure of the management leadership capability
3) $\forall s \in S \ : \ f(s)$ is the measure of the management conduction aptitude
4) $\forall s \in S \ : \ f(s)$ is the measure of the quality management potential
5) $\forall s \in S \ : \ f(s)$ is the measure of the management training knowledge
6) etc.

Let consider $\equiv_f$ be the equivalence relation on *S* induced by *f*

$$\forall s,t \in S \ : \ s \equiv_f t \Leftrightarrow f(s) = f(t)$$

Then, the equivalence class $(S / \equiv_f) = \{\pi(s) \mid s \in S\}$.

We will identify every partition $\pi(s)$ with the slice of a pie chart *PC* with the angle and the height determined by initial values associated with the function *f*, such that $PC = \bigvee_{i=1}^{|(S/\equiv_f)|} \pi_i(s)$.

The metric rating criterion was defined, and six grade rating levels were applied to these criteria. These levels are as follows: (1) very low, (2) low, (3) regular, (4) good, (5) very good, (6) excellent.

Each competence class in the prototype pie chart is calculated by the following formula:

$$\frac{\sum_{i=1}^{n} W_i C_i}{\sum_{i=1}^{n} W_i} \in [0,1]$$

where W_i is the weight of the competence and C_i is the expected value of the competence.

The approach degree is calculated by scoring the measured level and the expected level using the following formulas:

$$\text{Measured level} = \frac{\sum_{i=1}^{n} W_i M_i}{\sum_{i=1}^{n} W_i}$$

$$\text{Expectedlevel} = \frac{\sum_{i=1}^{n} W_i Cl_i}{\sum_{i=1}^{n} W_i}$$

$$\text{Approachdegree} = \frac{\sum_{i=1}^{n} W_i M_i}{\sum_{i=1}^{n} W_i Cl_i} \times 100$$

where W_i is the weight of the competence class, M_i is the value of the measure of the competence class, Cl_i is the expected value of the competence class and *n* is the number of the competence classes. Formulas are recursively applied for each level: competencies and sub-competencies.

The Utility Approach Degree is calculated extending (8) as following:

$$U_k^{\sigma_{hj}}(p_i) = \begin{cases} U_k(p_0) & if \quad i = 0, k = 0 \\ L(\alpha) * \left(\dfrac{\sum_{i=1}^{n} W_i M_i}{\sum_{i=1}^{n} W_i Cl_i} \right) & if \quad i > 0, k = 0 \,\&\, i \geq 0, k > 0 \\ U_k^{\sigma_{hj}}(p_i^{'}) * \left(\dfrac{\sum_{i=1}^{n} W_i M_i}{\sum_{i=1}^{n} W_i Cl_i} \right) & if \quad i > 0, k = 0 \,\&\, i \geq 0, k > 0 \end{cases}$$

where $\left(\dfrac{\sum_{i=1}^{n} W_i M_i}{\sum_{i=1}^{n} W_i Cl_i} \right)$ is the approach degree. The rest is as defined in the appendix A.

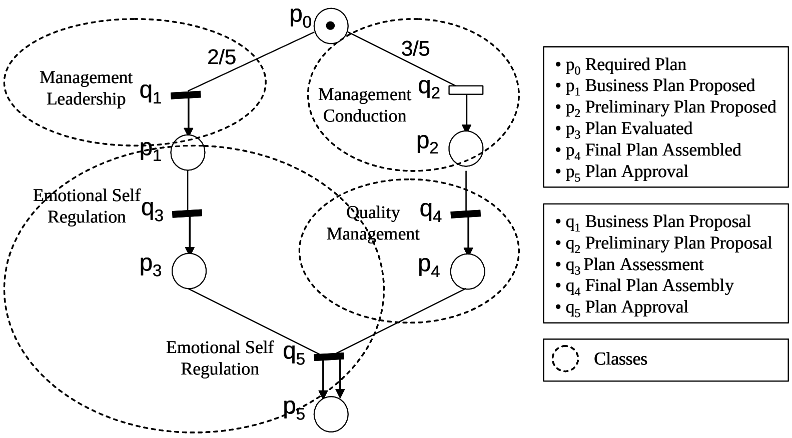

Fig. 8. Competency-Classes Prototype

4. Application example

In this section we present an application example. We define a scenario for a planning management. According to the process described in section three, we first design the business process. A section of the complete model of a budget plan is represented by an HDPPN in Fig. 8, Fig. 9 and Fig. 10 (the complete business model is out of the scope of this example). In the whole business-process model we identify 46 sub-competencies and following the methodology we make groups of 15 competencies and classify the competencies in five competency classes organized as follows:

1) Training Management
   a) Human resources skills
   b) Policy management
2) Quality Management
   a) Service-oriented management
   b) Quality control
   c) Change-oriented management

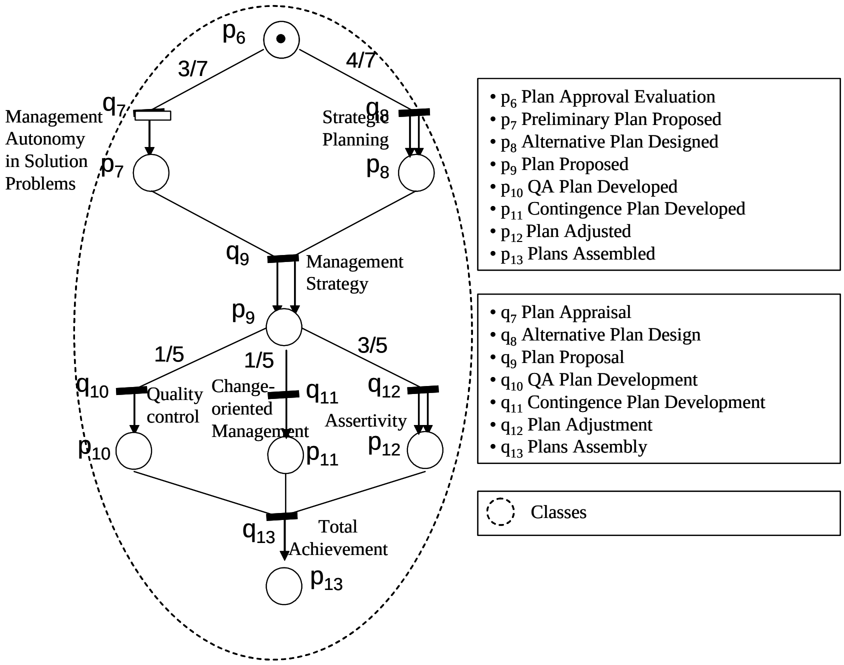

Fig. 9. Competency Prototype

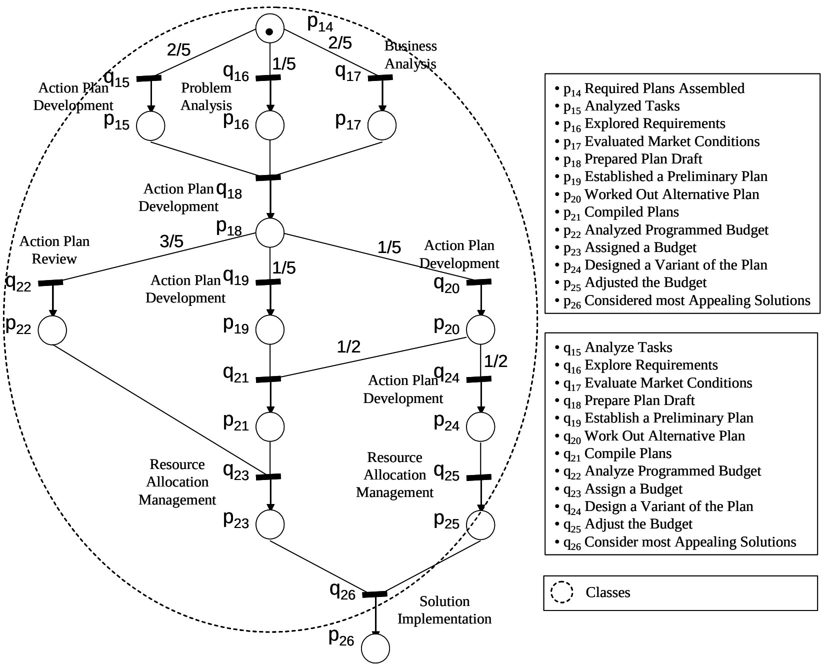

Fig. 10. Sub-Competency Prototype

   d) Total achievement
3) Leadership Management
   a) Assertiveness
   b) Negotiation
   c) Group management
4) Conduction Management
   a) Strategic planning
   b) Strategy management
   c) Autonomy in solution problems management
5) Emotional Self Regulation
   a) Adapting to stress
   b) Emotional language
   c) Confronting

Subsequently, we define the metrics to evaluate the employee prototype, and we select the psychological tools for rating the applicants. Tables in Fig. 11, Fig. 12 and Fig. 13 present examples of the competency refinement, and they resume the process.

From table in Fig. 12 and Fig. 13 we understand how to evaluate a sub-competency. In Fig. 12, we associate two psychological tools: the “Five Personality Factors” test and an “Interview”. Moreover, the “Five Personality Factors” test evaluates five factors, but we only choose the expressive sociability factor to evaluate the corresponding sub-competency. In Fig. 13, we use a complete test (Gnosis Facialis [Merten (2003)] or FEEL [Kessler et al. (2002)]) to evaluate the sub-competency.

Continuing with the process, for each sub-competency a score is achieved and represented in a pie chart. In the same way, competencies and classes of competencies are calculated and represented. In Fig. 14, Fig. 15 and Fig. 16 a real evaluation of 41 sub-competencies is shown. As a result, the model produces an approach-degree pie chart of how close the evaluated applicant is to what is expected from an ideal “employee prototype.”

Note that in Fig. 16, the result of the pie shows the opportunities areas (strengths and weaknesses) of the applicant. The measured result can be easily understood based in the following idea. If a slice of the resulted pie has a "white space," it represents a weaknesses area. Otherwise, the requirements are fulfilled. An applicant is ready to occupy a given position if the pie has no white spaces. That reveals how a development plan must be developed and applied. As a result, the development and the application order are determined by the weight and the average of the classes, competencies and sub-competencies.

For validation purposes, we calculate the utility of each sub-competency, competency and competency class, and weight the result with the percentage obtained in the appraisal step. The utility value obtained for each sub-competence will be weighted by the value obtained for the sub-competence in the appraisal step. The same process will be carried out for the calculation of the utility at each competency and competency class. Some utility calculations are as follows: define the Lyapunov-like function L in terms of the Entropy $H(p_i) = -p_i \ln p_i$ as $L = \max_{i=1,\ldots,|\alpha|} (-\alpha_i \ln \alpha_i)$ then we have

$$U_{k=0}(p_{14}) = 1 * 1$$

Class	Competence	Sub-Competence
Leadership Management	Assertivity	Communicate corporative needs
		Proceed straightforward in decision making
		Confront the critic
Emotional Self Regulation	Emotional Language	
		Emotional sensibility
		Emotional communication

Fig. 11. Sub-Competencies Definition

Sub-Competence	Definition
Communicating corporative needs	To communicate opportunely and clearly the corporative needs to superiors and subordinates
Metrics	① very low level efforts are required to communicate corporative needs ② low level efforts are required to communicate corporative needs ③ regular level efforts are required to communicate corporative needs ④ much effort is required to communicate corporative needs ⑤ too much effort is required to communicate corporative needs ⑥ highest efforts are required to communicate corporative needs
Measuring Tools	Five Personality Factors test. Dimension: expressive sociability factor Interview

Fig. 12. Sub-Competency Metrics and Measuring Tools

Sub-Competence	Definition
Emotional sensibility	The ability to identify in the others emotional states to take control of the situation
Metrics	① very low level ability is necessary to control situations ② low level ability is necessary to control situations ③ regular level ability is necessary to control situations ④ good level ability is necessary to control situations ⑤ very good level ability is necessary to control situations ⑥ excellent level ability is necessary to control situations
Measuring Tools	1) Gnosis Facialis (Merten, 2003) or FEEL (Kessler, 2002)

Fig. 13. Sub-Competency Metrics and Measuring Tools

the utility expected for places $\mathbf{p}_{15}$, $\mathbf{p}_{16}$ and $\mathbf{p}_{17}$ is as follows:

$$\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_{15}) = L[\sigma_{14,15}(p_{15}) * U_{k=0}^{\sigma_{14,15}}(p_{14})] * 0.75 = L[2/5 * 1] * 0.75$$
$$= \max H[2/5] * 0.75 = 0.366 * 0.75 = \mathbf{0.274}$$
$$\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_{16}) = L[\sigma_{14,16}(p_{16}) * U_{k=0}^{\sigma_{14,16}}(p_{14})] * 0.82 = L[1/5 * 1] * 0.82$$
$$= \max H[1/5] * 0.82 = 0.298 * 0.82 = \mathbf{0.244}$$
$$\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_{17}) = L[\sigma_{14,17}(p_{17}) * U_{k=0}^{\sigma_{14,17}}(p_{14})] * 0.98 = L[2/5 * 1]$$
$$= \max H[2/5] * 0.98 = 0.366 * 0.98 = \mathbf{0.358}$$

the utility expected for place $\mathbf{p}_{18}$ is as follows:

$$\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_{18}) = L[\sigma_{15,18}(p_{18}) * U_{k=0}^{\sigma_{15,18}}(p_{15}) + \sigma_{16,18}(p_{18}) *$$
$$U_{k=0}^{\sigma_{16,18}}(p_{16}) + \sigma_{17,18}(p_{18}) * U_{k=0}^{\sigma_{17,18}}(p_{17})] * 0.94$$
$$= L[2/5 * 0.366 + 1/5 * 0.298 + 2/5 * 0.366] * 0.94$$
$$= \max H[0.352] * 0.94 = 0.367 * 0.94 = \mathbf{0.344}$$

the utility expected for places $\mathbf{p}_{19}$ and $\mathbf{p}_{20}$ is as follows:

$$\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_{19}) = L[\sigma_{18,19}(p_{19}) * U_{k=0}^{\sigma_{18,19}}(p_{18})] * 0.91 = L[1/5 * 0.367] * 0.91$$
$$= \max H[1/5 * 0.367] * 0.91 = 0.191 * 0.91 = \mathbf{0.174}$$
$$\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_{20}) = L[\sigma_{18,20}(p_{20}) * U_{k=0}^{\sigma_{18,20}}(p_{18})] * 0.83 = L[1/5 * 0.367] * 0.83$$
$$= \max H[1/5 * 0.367] * 0.83 = 0.191 * 0.83 = \mathbf{0.158}$$

the utility expected for places $\mathbf{p}_{21}$ and $\mathbf{p}_{22}$ is as follows:

$$\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_{21}) = L[\sigma_{19,21}(p_{21}) * U_{k=0}^{\sigma_{19,21}}(p_{19}) + \sigma_{20,21}(p_{21}) * U_{k=0}^{\sigma_{20,21}}(p_{20})] * 0.78$$
$$= L[1 * 0.191 + 1/2 * 0.191] * 0.78$$
$$= \max H[0.253] * 0.78 = 0.358 * 0.78 = \mathbf{0.279}$$
$$\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_{22}) = L[\sigma_{18,22}(p_{22}) * U_{k=0}^{\sigma_{18,22}}(p_{18})] * 0.87 = L[3/5 * 0.367] * 0.87$$
$$= \max H[0.220] * 0.87 = 0.333 * 0.87 = \mathbf{0.289}$$

the utility expected for places $\mathbf{p}_{23}$, $\mathbf{p}_{24}$ and $\mathbf{p}_{25}$ is as follows:

$$\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_{23}) = L[\sigma_{21,23}(p_{23}) * U_{k=0}^{\sigma_{21,23}}(p_{21}) + \sigma_{22,23}(p_{22}) * U_{k=0}^{\sigma_{22,23}}(p_{22})] * 0.93$$
$$= L[1 * 0.358 + 1 * 0.333] * 0.93$$
$$= \max H[0.691] * 0.93 = 0.255 * 0.93 = \mathbf{0.237}$$
$$\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_{24}) = L[\sigma_{20,24}(p_{24}) * U_{k=0}^{\sigma_{20,24}}(p_{20})] * 0.96 = \max H[1/2 * 0.191] * 0.96$$
$$= 0.224 * 0.96 = \mathbf{0.215}$$
$$\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_{25}) = L[\sigma_{24,25}(p_{25}) * U_{k=0}^{\sigma_{24,25}}(p_{24})] * 0.79 = L[1 * 0.224] * 0.79$$
$$= \max H[0.224] * 0.79 = 0.335 * 0.79 = \mathbf{0.264}$$

the utility expected level of the sub-competencies defined in Fig. 10 for a given employee is:

$$\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_{26}) = L[\sigma_{23,26}(p_{26}) * U_{k=0}^{\sigma_{23,26}}(p_{23}) + \sigma_{25,26}(p_{26}) * U_{k=0}^{\sigma_{25,26}}(p_{25})] * 0.89$$
$$= L[1 * 0.255 + 1 * 0.335] * 0.89 = \max H[0.59] * 0.89$$
$$= \mathbf{0.311 * 0.89 = 0.277}$$

The utility expected level of the competencies related with the competencies in Fig. 9 is:

$$\begin{aligned}\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_7) &= L[\sigma_{6,7}(p_7) * U_{k=0}^{\sigma_{6,7}}(p_{26})] * 0.4 = L[3/7 * 0.311] * 0.4 \\ &= \max H[0.133] * 0.4 = \mathbf{0.268} * \mathbf{0.4} = \mathbf{0.107}\end{aligned}$$

The utility expected level of the corresponding competence class in Fig. 8 is:

$$\begin{aligned}\mathbf{U}_{k=0}^{\sigma_{hj}}(\mathbf{p}_2) &= L[\sigma_{0,2}(p_2) * U_{k=0}^{\sigma_{0,2}}(p_7)] * 0.33 = L[3/5 * 0.268] * 0.33 \\ &= \max H[0.160] * 0.33 = \mathbf{0.293} * \mathbf{0.33} = \mathbf{0.096}\end{aligned}$$

Concluding that for the competency-class Management Conduction, the utility value for this given employee will be 0.096. The ideal utility can be calculated using equation (6). Therefore, the validation process gives as a result an approach of the behavior of the proposed candidate on the business process model using the HDPPN.

Fig. 14. Classes-Evaluated Employee/Applicant

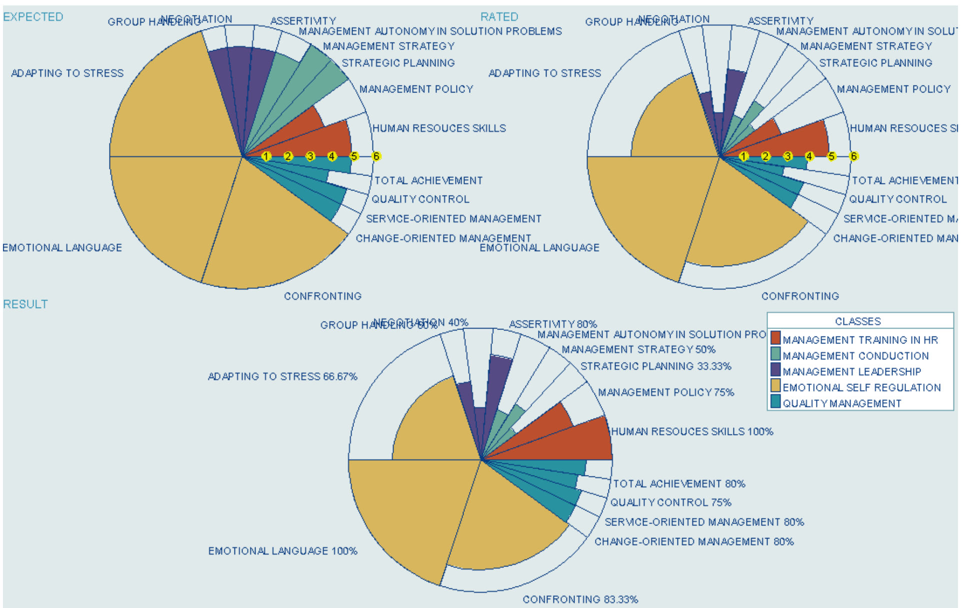

Fig. 15. Competency-Evaluated Employee/Applicant

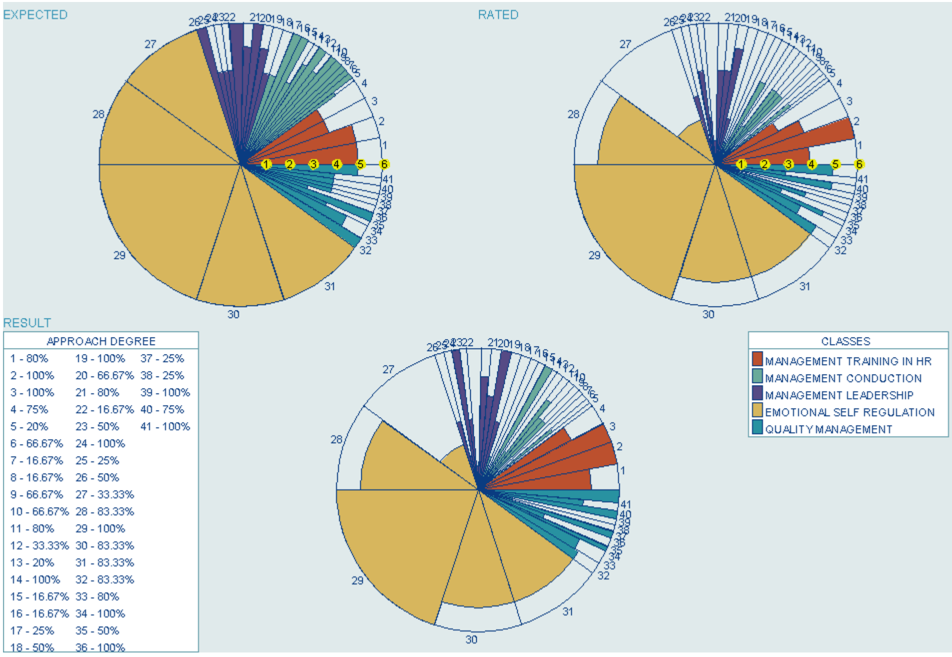

Fig. 16. Sub-Competency-Evaluated Employee/Applicant

5. Conclusion

A formal framework for human behavior modeling has been presented. The competencies model consists of a competencies-definition model, an evaluation-process model and a graphic-representation model. This model provides the necessary and desirable properties to develop the positions profiles. The model assists in addressing important issues in the evaluating area by helping to develop and quantify positions profiles. In addition, the competencies model offers the potential for progressively developing a knowledge base of competencies for selecting and evaluating job performance. As to issues concerning which competencies should be included, defined, and evaluated, every position profile is designed taking into account the business-process model. The modeling of the business process is based on business strategy transformation, establishing the relationship between the business strategy and the competencies definition using HDPPN. The competencies model provides a tool to define an employee prototype in terms of competencies, and it also provides a tool for determining how to evaluate such prototype. Psychological tools are used in the model to measure the competencies of the applicants. As a result, the model produces a "closeness degree" of how close the applicant is to what is expected from an ideal “employee prototype” in particular. To represent the positions profiles, the model is complemented with a special pie chart, where the weight of each competency is represented by the angle of the slice, and the achieved score is represented by its radius. The model provides all the information needed to build a development plan for an employee and how it must be developed and applied. The validation process simulates in HDPPN the possible behavior of each applicant. As a result, it produces a weighted utility value for each sub-competency, competency, and competency class at the corresponding HDPPN level, giving an approach of the behavior of the proposed applicant on the business-process model. This model can be applied to different application domains such as: education, health, crime and others. For instance, in the crime area, a suspect identification is performed inversely of what we previously did. That is, having a database of criminals, we would identify if the profile of a suspect (criminal prototype) is close to a criminal in the database. Current work related to the issues discussed in this example is concerned with the development of a software tool to support the design process, as well as the evaluation and the testing for large-scale industrial applications.

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Appendix A. Appendix

In this section, we present some well-established definitions and properties of HDPPN (see [Clempner (2010)]).

A.1. *Hierarchical Decision Process Petri Net*

Let $DPPN = \{P, Q, F, W, M_0, \pi, U\}$ be a Decision Process Petri net and let $f : P \cup Q \to 2^{P \cup Q}$ a refinement function such that $\forall s \in P \cup Q : f(s)$ defines the immediate descendant element of *s*.

Let $\equiv_f$ be the equivalence relation on $P \cup Q$ induced by *f* such that :

$$\forall s_1, s_2 \in P \cup Q : s_1 \equiv_f s_2 \Leftrightarrow f(s_1) =_f f(s_2)$$

then the collection of equivalence classes $(P \cup Q / \equiv_f) = \{\mathbf{C}(s) \mid s \in P \cup Q\}$ where $\mathbf{C}$ denotes class, is a poset. Thus, $(P \cup Q / \equiv_f)$ is linearly ordered and, consequently, it is a lattice. The structure $(P \cup Q / \equiv_f)$ is indeed trivial: all elements in $P \cup Q$ belonging to the same net under *f* are identified in this quotient set.

On the other hand, let us consider the relation $\leq_f$ as follows:

$$\forall s_1, s_2 \in P \cup Q : s_1 \leq_f s_2 \Leftrightarrow f(s_1) \leq_f f(s_2)$$

This relation is reflexive and transitive, but it is not antisymmetric in most cases[†]. Thus, $\leq_f$ is not an ordering in $P \cup Q$.

At this point let us recall some basic notions on orderings. A binary relation $\leq$ over a set X is a *partial order* if it satisfies the following three properties: reflexivity, antisymmetry and transitivity. A *total order* is a partial order that satisfies a fourth property known as comparability, where every element is related with every element one way or the other. A set and a partial order on that set define a *partially ordered set*, or *poset* for short. A *quasi order* is a relation $\leq$ that satisfies: reflexivity and transitivity. Formally, let $(X, \leq)$ be a poset and let $S \subseteq X$. Then an element $b \in S$ is a *minimal* element of *S* if there is no element $a \in S$ that satisfies $a \leq b$. Similarly an element $b \in S$ is a *maximal* element of *S* if there is no element $a \in S$ that satisfies $b \leq a$. It is

[†]It is antisymetric if and only if *f* is one-to-one

important to note about maximal elements is that they are in general not the greatest element of a subset *S*, formally we have an element $b \in S$ is the *greatest element* of *S* if for every element $a \in S$, $a \leq b$. Dually, an element $b \in S$ is the *least element* of *S* if for every element $a \in S$, $b \leq a$. Note that the least element of a poset is unique if one exists because of the antisymmetry of $\leq$. A *strict partial order* is a binary relation which is irreflexive, asymmetric and transitive. Strict partial orders correspond to directed acyclic graphs (DAGs), such that every strict partial order is a DAG, and the transitive closure of a DAG is both a strict partial order and also a DAG itself.

For any $s \in P \cup Q$ let the *successors* of *s*

$$t \in suc(s) \text{ iff } s \neq t, s \leq_f t \text{ and } \forall t_1 \ : \ s \leq_f t_1 \leq_f t \Rightarrow (t_1 =_f s) \vee (t_1 =_f t)$$

For any $s \in P \cup Q$ let the *predecessors* of *s*

$$t \in pre(s) \text{ iff } t \neq s, t \leq_f s \text{ and } \forall t_1 \ : \ t \leq_f t_1 \leq_f s \Rightarrow (t_1 =_f t) \vee (t_1 =_f s)$$

Therefore, let $P \cup Q$ be ordered by the following relationship:

$$\forall s_1, s_2 \in P \cup Q \, s_1 < s_2 \Leftrightarrow (s_1 <_f s_2) \vee (s_1 \equiv_f s_2) \vee (s_2 <_f s_1)$$

Thus, *f* is inducing a hierarchical structure on the *DPPN*.

Therefore, we can introduce the hierarchical partition $\{DPPN_\xi\}_{\xi \in \Xi}$ (where Ξ is a finite set) of the *DPPN* induced by *f*, such that each pair $(s,t) \in P_\xi \cup Q_\xi \ : (s,t)$ is an edge iff $t \in suc(s)$ in the $DPPN_\xi$ (or equivalently, $s \in pre(t)$). Let us say that *f* is consistent if the hierarchical structure has no cycles. From now on, we will consider only consistent functions.

A Hierarchical Decision Process Petri Net *HDPPN* is the graph whose set of nodes are the partition $\{DPPN_\xi\}_{\xi \in \Xi}$ induced by a refinement function *f*.

The *minimal elements* are those with no predecessors, i.e. nodes with null inner degree in *HDPPN*. The *maximal elements* are those with no successors, i.e. node with null outer degree in *HDPPN*.

Let us define the *upper distance* d^+ as follows:

$$d^+(s,t) = 1 \Leftrightarrow t \in suc(s)$$

$$d^+(s,t) = 1 + r \Leftrightarrow \exists t_1 \ : \ d^+(s,t_1) = r \ \& \ d^+(t_1,t) = 1$$

Similarly, the *lower distance* d^- is

$$d^-(s,t) = 1 \qquad \Leftrightarrow t \in pre(s)$$

$$d^-(s,t) = 1 + r \Leftrightarrow \exists t_1 \ : \ d^-(s,t_1) = r \ \& \ d^-(t_1,t) = 1$$

Thus, $d^+(s,t) = d^-(t,s)$.

The *upper height* of a node *s* is $h^+(s) = Max\{d^+(s_1,s) \mid s_1 \text{ is minimal}\}$. The *lower height* of a node *s* is $h^-(s) = Max\{d^-(s_1,s) \mid s_1 \text{ is maximal}\}$.

Let P_ξ, Q_ξ the set of places and transitions of the $DPPN_\xi$. Places and transitions in the *HDPPN* are numerated consecutively and will receive the number of the

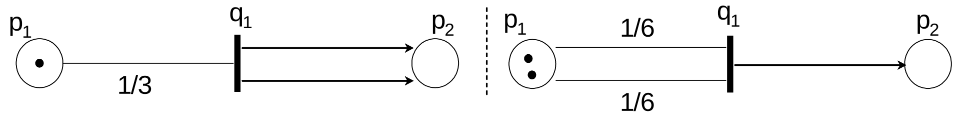

Fig. 17. Routing Policy case 1 Routing Policy case 2

corresponding $DPPN_{\xi}$ if will be necessary, i.e. $p_{\xi i}$ corresponds with the place *i* at the $DPPN_{\xi}$ otherwise we will identify the place only as p_i .

Let $M_{\xi k}(p_{\xi i})$ denote the marking (i.e., the number of tokens) at place $p_{\xi i} \in P_{\xi}$ at time k and let $M_{\xi k} = [M_{\xi k}(p_{\xi 1}),...,M_{\xi k}(p_{\xi m})]^T$ denote the marking (state) of $DPPN_{\xi}$ at time *k*. A transition $q_{\xi j} \in Q_{\xi}$ is said to be enabled at time *k* if $M_{\xi k}(p_{\xi 1}) \geq W_{\xi}(p_{\xi 1}, q_{\xi j})$ for all $p_{\xi i} \in P_{\xi}$ such that $(p_{\xi i}, q_{\xi j}) \in F_{\xi}$. It is assumed that at each time *k* there exist at least one transition to fire, i.e. it is not possible to block the net. If a transition is enabled then, it can fire. If an enabled transition $q_{\xi j} \in Q_{\xi}$ fires at time *k* then, the next marking for $p_{\xi i} \in P_{\xi}$ is given by

$$M_{\xi k+1}(p_{\xi i}) = M_{\xi k}(p_{\xi i}) + W_{\xi}(q_{\xi j}, p_{\xi i}) - W_{\xi}(p_{\xi i}, q_{\xi j}).$$

Let $A_{\xi} = [a_{ij}]$ denote a $n \times m$ matrix of integers (the incidence matrix) where $a_{ij} = a_{ij}^{+} - a_{ij}^{-}$ with $a_{ij}^{+} = W_{\xi}(q_{\xi i}, p_{\xi j})$ and $a_{ij}^{-} = W_{\xi}(p_{\xi j}, q_{\xi i})$. Let $u_k \in \{0,1\}^n$ denote a firing vector where if $q_{\xi j} \in Q_{\xi}$ is fired then, its corresponding firing vector is $u_k = [0,...,0,1,0,...,0]^T$ with the 1 in the j^{th} position in the vector and zeros everywhere else. The matrix equation (nonlinear difference equation) describing the dynamical behavior represented by a Petri net is:

$$M_{\xi k+1} = M_{\xi k} + A_{\xi}^T u_k$$

where if at step *k*, $a_{ij}^{-} < M_{\xi k}(p_{ij})$ for all $p_{\xi j} \in P_{\xi}$ then, $q_{\xi i} \in Q_{\xi}$ is enabled and if this $q_{\xi i} \in Q_{\xi}$ fires then, its corresponding firing vector u_k is utilized to generate the next step. Notice that if $M_{\xi}^{'}$ can be reached from some other marking M_{ξ} and, if we fire some sequence of *d* transitions with corresponding firing vectors $u_0, u_1, ..., u_{d-1}$ we obtain that

$$M_{\xi}^{'} = M_{\xi} + A_{\xi}^T u, \; u = \sum_{k=0}^{d-1} u_k .$$

In Fig. 17 we have represented partial routing policies π that generates a transition from state p_1 to state p_2 where $p_1, p_2 \in P$:

- case 1. The probability that q_1 generates a transition from state p_1 to p_2 is 1/3. But, because q_1 transition to state p_2 has two arcs, the probability to generate a transition from state p_1 to p_2 is increased to 2/3.

- case 2. We set by convention for the probability that q_1 generates a transition from state p_1 to p_2 is 1/3 (1/6 plus 1/6). However, because q_1 transition to state p_2 has only one arc, the probability to generate a transition from state p_1 to p_2 is decreased to 1/6.
- case 3. Finally, we have the trivial case when there exists only one arc from p_1 to q_1 and from q_1 to p_2.

Remark 1. In the previous definition we are considering nets with single initially marked place.

Remark 2. The previous definition in no way changes the behavior of the place-transitions Petri Net, the routing policy is used to calculate the utility value at each place of the net

Remark 3. It is important to note that the utility value can be re-normalized after each transition or time *k* of the net.

$U_k(.)$ denotes the utility at place $p_{\xi i} \in P_\xi$ at time *k* and let $U_k = [U_k(.),...,U_k(.)]^T$ denote the utility state of *HDPPN* at time k. $FN_\xi : F_\xi \to \mathrm{R}_+$ is the number of arcs from place p to transition q at level ξ (the number of arcs from transition q to place *p*). The rest of the *HDPPN* functionality is as described above.

Consider an arbitrary $p_{\xi i} \in P_\xi$ and for each fixed transition $q_{\xi j} \in Q_\xi$ that forms an output arc $(q_{\xi j}, p_{\xi i}) \in O_\xi$, we look at all the previous places $p_{\xi h}$ of the place $p_{\xi i}$ denoted by the list (set) $p_{\xi \eta_{ij}} = \{p_{\xi h} : h \in \eta_{ij}\}$ where $\eta_{ij} = \{h : (p_{\xi h}, q_{\xi j}) \in I \,\&\, (q_{\xi j}, p_{\xi i}) \in O\}$, that materialize all the input arcs $(p_{\xi h}, q_{\xi j}) \in I_\xi$ and form the sum

$$\sum_{h \in \eta_{ij}} \Psi(p_{\xi h}, q_{\xi j}, p_{\xi i}) * U_k(p_{\xi h})$$

where $\Psi(p_{\xi h}, q_{\xi j}, p_{\xi i}) = \pi(p_{\xi h}, q_{\xi j}) * \frac{FN_\xi(q_{\xi j}, p_{\xi i})}{FN_\xi(p_{\xi h}, q_{\xi j})}$ and the index sequence j is the set $\{j : q_{\xi j} \in (p_{\xi h}, q_{\xi j}) \cap (q_{\xi j}, p_{\xi i})$ & $p_{\xi h}$ running over the set $p_{\xi \eta_{ij}}\}$.

Proceeding with all the $q_{\xi j}$ s we form the vector indexed by the sequence *j* identified by $(j_0, j_1,..., j_f)$ as follows:

$$\left[\begin{array}{c} \sum_{h \in \eta_{ij_0}} \Psi(p_{\xi h}, q_{\xi j_0}, p_{\xi i}) * U_k(p_{\xi h}), \sum_{h \in \eta_{ij_1}} \Psi(p_{\xi h}, q_{\xi j_1}, p_{\xi i}) * U_k(p_{\xi h}),..., \\ \sum_{h \in \eta_{ij_f}} \Psi(p_{\xi h}, q_{\xi j_f}, p_{\xi i}) * U_k(p_{\xi h}) \end{array} \right]$$

Intuitively, the vector (15) represents all the possible trajectories through the transitions $q_{\xi j}$ s where $(j_1, j_2,..., j_f)$ to a place $p_{\xi i}$ for a fixed *i* and ξ.

Continuing the construction of the definition of the utility function *U*, let us introduce the following definition.

Let $L : \mathbb{R}^n \to \mathbb{R}_+$ be a continuous map. Then, L is a Lyapunov-like function (see [Kalman and Bertram (1960)]) iff satisfies the following properties:

1. $\exists x^*$ such that $L(x^*) = 0$,
2. $L(x) > 0$ for $\forall x \neq x^*$,
3. $L(x) \to \infty$ when $x \to \infty$,
4. $\Delta L = L(x_{i+1}) - L(x_i) < 0$ for all $x_i, x_{i+1} \neq x^*$.

Then, formally we define the utility function U as follows:

Let *HDPPN* a Hierarchical Decision Process Petri Net. The utility function U is represented by the equation

$$U_k^{q_{\xi j}}(p_{\xi i}) = \begin{cases} U_k(p_0) & if \;\; i = 0, k = 0 \\ L(\alpha) & if \;\; i > 0, k = 0 \,\&\, i \geq 0, k > 0 \\ U_k^{q_{\xi j}}(p'_{\xi i}) & if \;\; i > 0, k = 0 \,\&\, i \geq 0, k > 0 \end{cases}$$

where

$$\alpha = \begin{bmatrix} \sum\limits_{h \in \eta_{ij_0}} \Psi(p_{\xi h}, q_{\xi j_0}, p_{\xi i}) * U_k^{q_{\xi j_0}}(p_{\xi h}), \sum\limits_{h \in \eta_{ij_1}} \Psi(p_{\xi h}, q_{\xi j_1}, p_{\xi i}) * U_k^{q_{\xi j_1}}(p_{\xi h}), \ldots, \\ \sum\limits_{h \in \eta_{ij_f}} \Psi(p_{\xi h}, q_{\xi j_f}, p_{\xi i}) * U_k^{q_{\xi j_f}}(p_{\xi h}) \end{bmatrix}$$

the place $p'_{\xi i} \in f(p_{\xi i})$ is the initial marked place of the $DPPN_{\xi}$, the function $L : D \subseteq \mathbb{R}_+^n \to \mathbb{R}_+$ is a Lyapunov-like function which optimizes the utility through all possible transitions (i.e. through all the possible trajectories defined by the different $q_{\xi j}$ s), D is the decision set formed by the j's ; $0 \leq j \leq f$ of all those possible transitions $(q_{\xi j}, p_{\xi i}) \in O$, $\Psi(p_{\xi h}, q_{\xi j}, p_{\xi i}) = \pi(p_{\xi h}, q_{\xi j}) * \frac{FN(q_{\xi j}, p_{\xi i})}{FN(p_{\xi h}, q_{\xi j})}$, η_{ij} is the index sequence of the list of previous places to $p_{\xi i}$ through transition $q_{\xi j}$, $p_{\xi h}$ $(h \in \eta_{ij})$ is a specific previous place of $p_{\xi i}$ through transition $q_{\xi j}$.