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OPTIMISED EIGENSTRUCTURE ASSIGNMENT BY ANT SYSTEM AND LQR APPROACHES

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Recently, it has been verified that applications of metaheuristics for finding optimal or suboptimal solutions for NP-hard optimisation problems is one of the most promising research fields. Using the ant system metaheuristic, this paper proposes a new design method for state feedback control law which simultaneously achieves the linear quadratic stabilisation and an eigenstructure assignment corresponding to partial closed-loop eigenvalues and eigenvectors. As a typical application of the method, flight control design for an aircraft state space model is presented to illustrate the application of the Ant System and LQR method, the purpose of which is to find a state feedback controller that leads to a specified eigenstructure assignment.

Keywords: Ant System; LQR method; Eigenstructure Assignment; Optimised Control Law; Flight Control Application.

1. Introduction

Control theory has been a classical conceptualisation of feedback and control of the physical system, from an engineering point view. This theory needs a rigorous definition of many mathematically complex tools. However, different classical and robust control algorithms have been proposed for the optimisation control law computing. The most common ones are LQR, LQG, H_2, H_∞ control and eigenstructure assignment [Chiappa (1998), Kubika and Livet (1994), Kucera (1986), Kucera (1992), Sobel and Shapiro (1985), Tsui (2001)].
But recently, it has been verified that the approaches issued from the soft computing such as Fuzzy Logic, Genetic Algorithms, Neural Network and Ant System are simple, practical, adaptable and computationally efficient to solve several practical optimisation problems including hard industrial objectives. Moreover, the infinite horizon linear quadratic regulator (LQR) and the eigenstructure assignment (EA) are among the most popular controller design techniques for MIMO (Multi Input Multi Output) systems. The advantage of EA is that if the specifications are given in terms of system eigenstructure,

the eigenstructure can be achieved exactly for the desired stability and dynamic performance. However, the EA suffers from some limitations as that the system performance may not be optimised in some practical cases, such as minimum control effort, and that the system requirements are often not easily specified in terms of eigenvalues/vectors. Furthermore, the LQR could be used to optimise the controller design by minimizing a quadratic cost function of system response and control energy. For deterministic system, the LQR-based control design generally guarantees the closed-loop stability and certain degrees of robustness, but may not easily achieve specific system performances due to the difficulty in the selection of the synthesis matrices Q and R.

Hence, via the EA techniques, the choice of the LQR design matrices Q and R presents a nice problem to be studied by applying the efficient heuristics and methods inspired from Nature's Laws, especially the ant system optimisation metaheuristic.

Initially, submitted to application by Dorigo et *al* [Colorni et al. (1992), Davis and Clark (1995), Stützle and Dorigo (2002)], the ant system optimisation presents a class of general algorithms of optimisation. The main underlying idea, essentially inspired from the behaviour of real ants, represents a parallel search of several constructive computational solutions. These latter are based on the problem characteristic data and on a dynamic structure memory containing information on the quality of previous solutions. Moreover, the ant system metaheuristic has been successfully applied to a variety of combinatorial optimization problems such as the travelling salesman problem and different variants of the scheduling problem. Convergence proofs for the Ant Colony Optimization algorithms can be found in [Gutjahr (2003), Stützle and Dorigo (2002)].

This paper is organised as follows: Section 2 is devoted to the presentation and mathematical formulation of the EA and the LQR problems. In Section 3, a general framework of the proposed optimisation procedure will be illustrated based on the ant system metaheuristic. The experimental framework and the computational results will be presented in Section 4

2. Eigenstructure Assignment and LQR

Eigenstructure Assignment concept allows the designer to assign the closed-loop eigenvalues and also some or all eigenvectors depending on the application requirement. By the assignment of eigenvectors, the zeros of the transfer functions can be influenced and coupling and decoupling modes can be addressed directly. Although the standard technique takes into account the system performance and the decoupling modes, it does not address the optimisation problem. Moreover, the LQR could be used to compute an optimal controller by minimizing a quadratic cost function of system response and control energy. However, the LQR technique may not easily achieve a specific system performance due to ambiguities in the selection of the synthesis weighting matrices, Q and R, in the optimisation process. This problem must be addressed by any control design strategy, and has been considered within many practical cases. In this work, the Ant System optimisation metaheuristic can be used to search the weighting matrices Q and R

with Q is a [n x n] matrix and R is a [m x m] matrix. The design problem is described as follows:
Given a state variable model of the form:

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (1)$$

Where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control vector, $A \in R^{n\times n}, B \in R^{n\times m}$ are constant matrices, such that (A, B) is a controllable pair and B has full column rank.
The LQR is produced by the minimization of the following cost function:

$$J = \int_0^\infty (x^T Q\, x + u^T R\, u)dt \qquad (2)$$

in which $Q \geq 0$ and $R > 0$ that defining respectively, the state and the input weighting matrices of the LQR optimisation problem.
According to the LQR technique, the system (Eq. (1)) is controlled by the state feedback

$$u = K_R^Q x \qquad (3)$$

The closed-loop system representation is given by:

$$\dot{x}(t) = A_c x(t) = (A + BK_R^Q)x(t) \qquad (4)$$

Via the following Algebric Ricatti Equation *(ARE)*

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad (5)$$

we can determine the solution *P* and the gain K_R^Q

$$K_R^Q = R^{-1}B^T P \qquad (6)$$

A suitable choice of the *Q* and *R* leads to the computation of the gain K_R^Q. Therefore, the eigenvalues and eigenvectors of the closed-loop system can be chosen and the system performances are provided.
The application of the Ant System metaheuristic represents the main idea to determine the weighting matrices *Q* and *R*.
The weighting matrices *Q* an *R* may be manipulated by the Ant System metaheuristic in order to force the system to behave in the required manner by the designer, in this case to force an eigenstructure upon the closed loop system. Provided that the controller is a solution of the Algebric Ricatti Equation (Eq. (5)), then the performance and robustness of the LQR will be maintained.
The main aim is to setup a cost function relating the desired eigenstructure to the eigenstructure achieved by the present solution under consideration. The cost function value is derived from the difference between desired and achieved eigenstructures, determined by their eigenvalues $\{\lambda_{di}, \lambda_{ai}\}$ and their corresponding eigenvectors $\{v_{di}, v_{ai}\}$, $\forall\, i = 1, 2, \ldots n$.
For this, we define the following criteria:

$C_1 = \underset{i}{Max}\{|\lambda_{di} - \lambda_{ai}|\}, \forall\, i = 1, 2, ...n$, the maximum norm of the difference between the desired and achieved eigenvalues.

$C_2 = \underset{i}{Max}\{\|v_{di} - v_{ai}\|^2\}, \forall\, i = 1, 2, ...n$, the Euclidean distance between $(\vec{v}_{di})$ and $(\vec{v}_{ai})$.

The minimization of the Cg = $Max\{C_1, C_2\}$ criterion, shows that the following cost function can be used to search an optimized eigenstructure assignment by ant system and LQR method. Then, our objective is to minimise the global criterion Cg defined by:

$$C_g = Max\{C_1, C_2\} \qquad (7)$$

Thus, a zero cost function value represents the ideal solution.

The minimisation of the cost function is achieved through manipulation of these Q and R matrices by the ant system metaheuristic.
Based on the above criteria, an algorithm for the optimised closed loop eigenstructure assignment can be give.

3. Ant System optimisation via LQR method (*AS-LQR*)

In our case, the design of the optimised closed loop eigenstructure assignment is based on the Ant System metaheuristic [Kubica and Livet (1994), Kucera (1986)].
The AS imitates the way of real ants [Kucera (1992), Davis and Clarck (1995), Wang et al. (2007)] to find the shortest route between a food source and their nest. The ants communicate with each other by means of pheromone trails and exchange information about which path should be followed. The more the number of ants traces a given path, the more attractive this path becomes and is followed by other ants by depositing their own pheromone. This collective ant system behaviour results in the establishment of the shortest route.
The method improved by modeling real ant behavior uses exactly the following specifications:

- The communication established with ants through pheromone trail.
- Paths deposited more pheromone preferred previously.
- Pheromone trail on short path increase more rapidly.

The main idea of the AS-LQR metaheuristic is presented below:

Initialise Ant System parameters
While (until the termination criteria are met) do
Generate Promising Solutions (Qi and Rj)
Apply the LQR(Qi,Rj)
Compute the criteria C1 and C2.
Evaluate solution minimising the C_g criterion
If a *new solution* is improved, **then**

The current *best solution* becomes *new solution*
Save the current given best solution in tabu list
Update Pheromone Trail
End

Initially, each ant is placed on a *Begin* node and a next Q_i, R_j *and* the *End* nodes are randomly chosen. The chosen nodes represent a solution path given by the initial exploration space search.

Typically, ants deposit the chemical pheromone when they move around in their environment; they are also able to detect and follow pheromone trails.

In our case, the pheromone trail describes how the ant builds the optimized solution of the Ant System problem. On the construction graph (Figure 1), the probability of choosing a Q_i node and Rj node depends on the total amount of pheromone on the node which is proportional to the number of ants visiting the node at the *k-th* iteration algorithm.

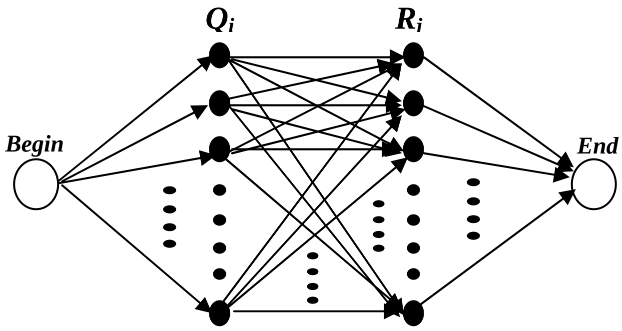

Fig. 1 The construction graph modelling of the Q_i and R_j matrices choice

The $P^a_{i,j}$ represents the probability of the *a-th* ant to use the Q_i and R_j for the *LQR* computing optimisation. Each ant builds a solution using a combination of the information provided by the pheromone trail $\tau_{i,j}$ and the heuristic functions which are defined by $\eta_i = \|Q_i\|_\infty$ and $\mu_j = \|R_j\|_\infty$ of the Q_i and R_j matrices.

$$P^a_{i,j} = \begin{cases} \dfrac{(\tau_{i,j})^\alpha (\eta_i)^{\beta_1} (\mu_j)^{\beta_2}}{\sum_{i,j \in D} (\tau_{i,j})^\alpha (\eta_i)^{\beta_1} (\mu_j)^{\beta_2}} & if\,(i,j) \in D \\ 0 & otherwise \end{cases} \qquad (8)$$

where $\eta_i = \|Q_i\|_\infty$ and $\mu_j = \|R_j\|_\infty$ represent respectively the infinite norm.

In equation (Eq. (8)), D denotes the set of nodes visited by the *a-th* ant, where α, β_1 and β_2 are the parameters which control the relative importance of the pheromone trail versus heuristic. Therefore, the transition probability is a trade-off between heuristic consequence and pheromone trail intensity at a given time.

3.1. *Pheromone Update*

To allow the ants to share information about good solutions, the updating of the pheromone trail must be established. After each iteration of the ant system algorithm, equation (Eq. (9)) describes in detail the pheromone update used when each ant has completed its own optimised feedback controller solution S^{ant} characterized by the criterion $C_g = C^{ant}$ according to the equation (Eq. (7)). This latter corresponds to the length tour T^{ant}. In order to guide the ant system toward good solutions, a mechanism is required to assess the quality of the best solution. The obvious choice would be to use the iteration-best criterion C^{min} of all solutions given by a set of ants at the current iteration:

$$\Delta\tau_{i,j}^{a} = \begin{cases} \dfrac{C^{\min}}{C^{ant}} & if\,(i,j)\in T^{ant} \\ 0 & otherwise \end{cases} \qquad (9)$$

After all the ants have completed their tours, the trail levels on each node need to be updated. The evaporation factor ρ ensures that the pheromone is not accumulated infinitely and indicates the quality of the pheromone that is approved over to the next algorithm iteration. Equation (Eq. (10)) represents the pheromone-level-update:

$$\tau_{i,j} = (1-\rho)\,.\,\tau_{i,j} + \sum_{1}^{N^a} \Delta\tau_{i,j}^{a} \qquad (10)$$

Where N^a defines the number of ants to use in the ant system, and ρ is the pheromone parameter which decays the pheromone trail.

3.2. *The setup parameter values*

The setup parameter values used in the Ant System optimised controller are often very important in getting good results. However, the exact values are very often entirely problem dependent, and cannot always be derived from features of the problem itself.

- α determines the degree to which pheromone trail is used as the ants build their solution. The lower values indicate that the pheromone trail has no important role. Moreover, the higher values imply that the ants perform too little exploration. After test phase, the algorithm works well with relatively high values [0.4…0.8].
- β_1 and β_2 determine the extent to which heuristic information is used by the ants. Again, values between 0.3 and 0.6 appeared to offer the best trade-off between the effect of the heuristic and the research space exploration.
- τ_0 is the value to which the pheromone trail values are initialized. Initially, the value of the parameter should be moderately high to encourage initial exploration, while the pheromone evaporation procedure will gradually stabilize the pheromone trail.
- ρ is the pheromone evaporation parameter and is always set to be in the range $0<\rho<1$. It defines how quickly the ants 'forget' past solution. A higher value makes for a more aggressive search. A value test around 0.4 to 0.8 leads to find good solutions.
- N^a defines the number of ants to use in the colony, a low value speeds up the algorithm because few searches are done, a high value slows the search down, as

more ants run before each pheromone update is performed. A value of 10 appears to be a good compromise between the execution speed and the quality of the solution achieved.

4. Benchmark and simulation results

To illustrate the effectiveness and performance of the proposed approach in this paper, a representative practical example taken from the literature can be presented. For more information, see references (Davis and Clarck (1995)) and (Sobel and Shapiro (1985)). The system describes a Lockheed aircraft, L1011 Tristar type and the A, B and C matrices are given below.

$$x = \begin{bmatrix} \partial_r & : Rudder\ deflexion(rad) \\ \partial_a & : Aileron\ deflexion(rad) \\ \phi & : Bank\ angle(rad) \\ r & : Yaw\ rate(rad/s) \\ p & : Roll\ rate(rad/s) \\ \beta & : Sideslip\ angle(rad) \end{bmatrix} \quad u = \begin{bmatrix} \partial_{rc} & : Rudder\ command(rad) \\ \partial_{ac} & : Aileron\ command(rad) \end{bmatrix}$$

$$A = \begin{bmatrix} -20 & 0 & 0 & 0 & 0 & 0 \\ 0 & -25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -0.744 & -0.032 & 0 & -0.154 & -0.0042 & 1.54 \\ 0.337 & -1.12 & 0 & 0.249 & -1 & -5.20 \\ 0.02 & 0 & 0.0386 & -0.996 & -0.00029 & -0.117 \end{bmatrix} \quad B = \begin{bmatrix} 20 & 0 \\ 0 & 25 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For this implementation example, the different results obtained by the proposed approach are presented and compared with the other results found in (Davis and Clarck (1995)).
In fact Davis R. and Clarck T. consider in their Genetic Algorithm procedure that:

- Roll Subsidence Mode time constant τ_r must not exceed 1.3s. This translates to a roll subsidence mode eigenvalue λ_r, which must not exceed -0.769. For this example a value of -2.0 was used.

- Spiral Mode instability is allowable, but the time to double starting at 30° bank angle must not be quicker than 20s. The LQR produces a spiral mode which is stable and, hence, this mode was not set a specific value, but was checked on completion for compliance with the requirements.

Dutch Roll requirements are presented in terms of frequency and damping of the second-order mode. ξ_d must be ≥ 0.4, $\omega_{0d} \geq 1$ and the product $\xi_d \omega_{0d} \geq 0.4$. For the example the values $-1.5 \pm 1.5j$ were selected for the Dutch Roll eigenvalues witch corresponds to $\xi_d = 0.707$ and $\omega_{0d} = 2.12$.

The resultant gain matrix, K^{PI-GA}, produced by G.A. implementation, is as follows:

$$K^{PI-GA} = \begin{bmatrix} 0 & 0 & -0.1169 & -3.2996 & -0.4584 & 2.8277 \\ 0 & 0 & -0.2947 & -0.9645 & -0.6826 & 3.8995 \end{bmatrix}$$

The desired and achieved eigenvalues, produced by G.A. implementation, is as follows:

$$\lambda_d \begin{bmatrix} -25 \\ -20 \\ -1.5+1.5j \\ -1.5-1.5j \\ -2 \\ x \end{bmatrix} \qquad \lambda_a = \begin{bmatrix} -24.201 \\ -17.342 \\ -1.427+1.539j \\ -1.427-1.539j \\ -1.630 \\ -0.243 \end{bmatrix}_{PI-GA}$$

All results of the optimized Eigenstructure Assignment by the Ant System and the LQR method (*AS-LQR*) are presented for 1000 iterations with 10 the number of ants, and each run was performed 10 times. The algorithms have been coded in Matlab7 tools and tested using a P4 Pentium processor 2.4 GHz.
With $N^a = 10; \alpha = 0.4; \ \beta_1 = 0.5; \beta_2 = 0.5; \tau_0 = 0.7; \rho = 0.5$,
The resultant gain matrix, K^{AS-LQR}, produced by Ant System and LQR (AS-LQR) implementations, is as follows:

$$K^{AS-LQR} = \begin{bmatrix} 0 & 0 & 0.0291 & -3.3420 & 0.0013 & 2.2714 \\ 0 & 0 & -0.5305 & -0.5276 & -1.7292 & 3.5525 \end{bmatrix}$$

The desired and achieved eigenvalues, produced by AS-LQR implementation, is as follows:

$$\lambda_d \begin{bmatrix} -25 \\ -20 \\ -1.5+1.5j \\ -1.5-1.5j \\ -2 \\ x \end{bmatrix} \qquad \lambda_a = \begin{bmatrix} -22.454 \\ -16.949 \\ -1.518+1.507j \\ -1.518-1.507j \\ -2.958 \\ -0.874 \end{bmatrix}_{AS-LQR}$$

Dutch Roll exhibits a damping of 0.68 and a natural frequency 2.1 rad/s for the G.A. implementation and a damping of 0.71 and a natural frequency 2.14 rad/s for the AS-LQR implementation. The spiral Mode is stable and the time constant τ_r is 0.61s for G.A. implementation and 0.34s for the AS-LQR implementation, which is well below the 1.3s. λ_1 *and* λ_2 (the actuator poles) both show values less negative for G.A. (-24.201, -17.342) and AS-LQR (-22.454,-16.949) implementations than their open-loop values (-25,-20). The actuator bandwidths are therefore not exceeded by the closed-loop controller.

The desired and achieved eigenvectors, necessary for mode decoupling, produced by G.A. implementation, are given by:

$$v_{3,4.d} \begin{bmatrix} x+xj \\ x+xj \\ 0+xj \\ 1+xj \\ 0+xj \\ x+j \end{bmatrix} \quad v_{3,4.a} = \begin{bmatrix} -0.766 \mp 0.259j \\ -0.186 \mp 0.416j \\ 0.001 \mp 0.070j \\ -0.207 \mp 0.221j \\ 0.106 \mp 0.100j \\ 0.019 \mp 0.140j \end{bmatrix}_{PI-GA} \qquad v_{5.d} \begin{bmatrix} x \\ x \\ x \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad v_{5.a} = \begin{bmatrix} 0.690 \\ 0.156 \\ -0.613 \\ 0.206 \\ 1 \\ 0.142 \end{bmatrix}_{PI-GA}$$

The desired and achieved eigenvectors, necessary for mode decoupling, produced by AS-LQR implementation, are given by:

$$v_{3,4.d} \begin{bmatrix} x+xj \\ x+xj \\ 0+xj \\ 1+xj \\ 0+xj \\ x+j \end{bmatrix} \quad v_{3,4.a} = \begin{bmatrix} 0.861 \\ -0.112 \mp 0.247j \\ 0.058 \mp 0.105j \\ 0.273 \mp 0.104j \\ -0.247 \mp 0.071j \\ 0.047 \mp 0.122j \end{bmatrix}_{AS-LQR} \qquad v_{5.d} \begin{bmatrix} x \\ x \\ x \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad v_{5.a} = \begin{bmatrix} 0.585 \\ 0.647 \\ -0.141 \\ 0.193 \\ 0.420 \\ 0.065 \end{bmatrix}_{AS-LQR}$$

Inspection of these vectors shows that the desired decoupling is present for the two implementations. System robustness may be expressed as the minimum singular value of the return difference matrix (Garg (1991)). The minimum here, overall frequencies is 0.896, which corresponds to a guaranteed phase margin of 53° for GA implementation and overall frequencies is 0.901, which corresponds to a guaranteed phase margin of 56° for AS-LQR implementation.

The figure 2 represents simulation results of the Lockheed aircraft, L1011 Tristar type system responses for impulsional signal with a PI-GA and AS-LQR controller. In fact figure 2 shows clearly reduced time response less than 20 iterations for AS-LQR controller and around 40 iterations for PI-GA controller, an overshoot less than 7% for AS-LQR controller and around 18% for PI-GA controller.

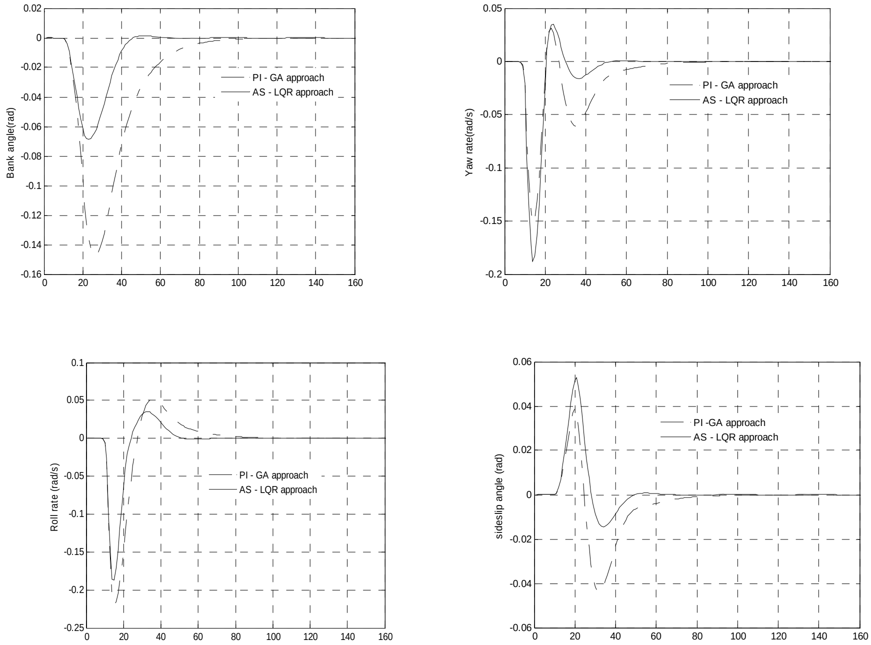

Fig. 2. Evolution of the states variables for the system using AS-LQR and PI-GA approaches.

The Figure3 represents the pheromone trail evolution on the construction graph composed by 50 Q_i nodes and 50 R_j nodes.

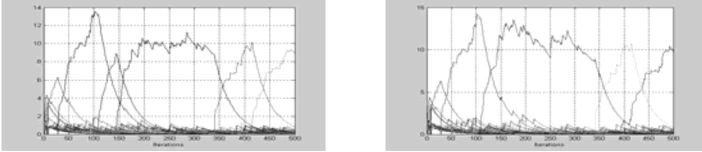

Fig. 3. Sample run of pheromone trail evolution on Qi nodes and Rj nodes

The figure 4 represents a sample run of the cost function evolution related to the construction graph given in figure3 and we remarks that it converges in 145 iterations which corresponds to a reasonable computing time.

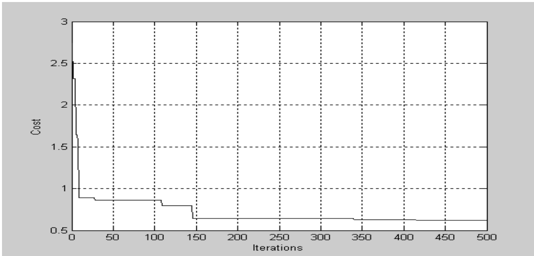

Figure 4. Sample run of cost function evolution

5. Conclusions:

The design of an optimised Eigenstructure Assignment using the Ant System metaheuristic in conjunction with a linear-quadratic regulator (*LQR*) control law theory for time invariant systems is presented. The design methodology needs the minimisation of a cost function which relates the desired and achievable eigenstructure given by the *AS-LQR* approach. Essentially the *LQR* could be used to optimise the controller design by minimizing a quadratic cost function of system response and control energy. The Ant System metaheuristic manipulates the matrices Q and R of the *LQR* procedure until a desired eigenstructure performance is achieved. Empirically, the method works well and leads to a good solution in reasonable time.

The algorithm was applied within a practical flight control system framework and produced an output feedback controller which satisfies the design requirements. The efficiency of the proposed *AS-LQR* for an optimised Eigenstructure Assignment is evaluated and compared with the results obtained from parallel implementation of genetic algorithm methods *PI-GA*. The results of the *AS-LQR* implementation are reasonable and closer to the design requirements with a guaranteed level of robustness.

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