# How Placing Limitations on the Size of Personal Networks Changes the Structural Properties of Complex Networks

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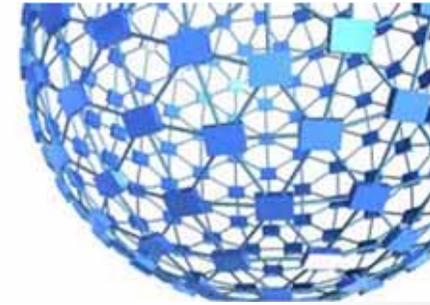Seoul National University
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1

Table of content

- Introduction
  - Problem Description
  - Research Question
  - Research Contribution
- Theoretical Background
- Proposed network formation model
- Experimental Results
- Conclusion
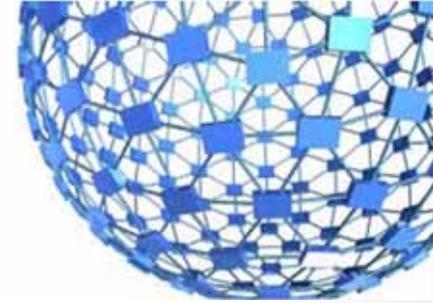
Introduction

- The person-to-person interactions between all of the individuals in our society form a large social network of people with interesting features (i.e., six degree of separation).

- These days, such interactions have been extended through **social media** and **virtual communities**

  - Recent studies have shown that the **six degrees of separation has shrunk**, due to social networking tools (e.g., Facebook).

- Existing research on "small world" theory focused only on calculating the average shortest path length of networks that utilize all the existing connections among people [1, 6].

Problem Description
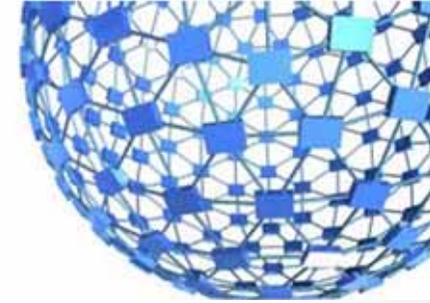
- We can argue that the technology is shrinking our world. **However, the maintenance of our personal network is not costless.** It requires **spending time** and **effort** to maintain relationships.

- Two types or relationships can be distinguished:
  - Talking about people you **know** is about revealing your social circle in the society and the people you meet in your daily life.
  - Talking about people you **trust** is about revealing your personal preferences in different contexts.

- If we want to forward **critical information** from a start point to a target point in another part of the network, we cannot utilize all the existing connections.

- **There is a limit on how many close friends one can have** compared to known people.

Research Question

- We ask
  - *how a limitation in the number of trusted contacts impacts the structural properties of the entire network.*

- We investigate
  - the extent, to which a limitation on the size of personal networks (i.e., having a few trusted friends only) leads to an increase in the average shortest path length of the entire network.

5

Research Contribution

- We propose the following features that a network growth model should incorporate:
  - **Variability in individual patterns of behavior:**
    - The establishment of links of different types needs to be considered due to the difference of behavior of people in social environments.
  - **Different rate of variability of new node entrance and link establishments:**
    - The rate, at which people join a network, is different than the rate of link creation among existing ones.
  - **Limitations on the size of personal networks:**
    - The number of trusted contacts is smaller than the number of known contacts.
- Proposed features distinguish our analysis of the network's average path length from previous analyses in literature.

Theoretical Background
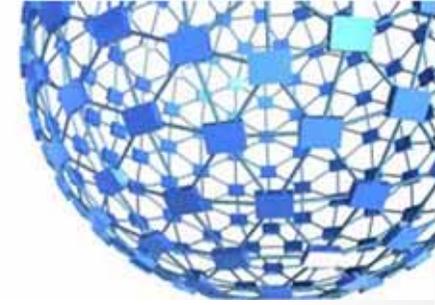
- Stanley Milgram, in his famous experiments [3, 4], was interested in **computing the distance distribution of the acquaintance graph**.
  - The famous small world theory of personal relationships comprises the idea of being connected to any other person by a chain of only five people in average (i.e., six degree of separation).

- Backstrom et al. repeated Milgram's experiment by using the entire Facebook network and reported the observed average distance of 4.74, corresponding to 3.74 intermediaries or "degree of separation" [1].

- **The small world theory has been criticized with respect to some issues (e.g., low success rate in some experiments) [5].**

Proposed Network Formation Model
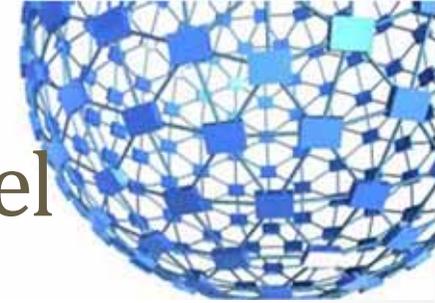
- We conduct a multi-agent-based simulation in Netlogo [13]

- The **proposed model is a generative model** based on the ideas that individual pattern of behavior in social environment is the key.

  - This model can generate a network, in which the members **follow the classical preferential attachment** (i.e., attractiveness of each individuals is modeled by existing attachments [15, 16]) for connecting to other users.

  - Furthermore, **the users in this model have the ability to create potential links**.

Proposed Network Formation Model
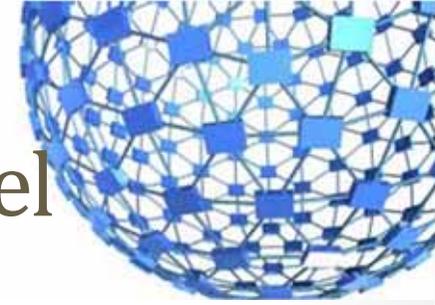
- Interactions among people of a network can be regarded as **dynamic processes**, **which lead to changes in its structural properties**.

- Dynamic processes in turn can be categorized into two groups:
    - (1) **the process, which occurs during the growth of a network** and represents the tendency of new users to establish links to other members upon entry into the network;
    - (2) **the process, which occur among existing users of a network** in order to establish links between them.
        - (a) the first category includes the **establishment of potential links of a Friend-Of-A-Friend type (FOAF type)** with social distance equal to 1; and
        - (b) the second category includes the **establishment of random potential links** with other users, who are present in the network.
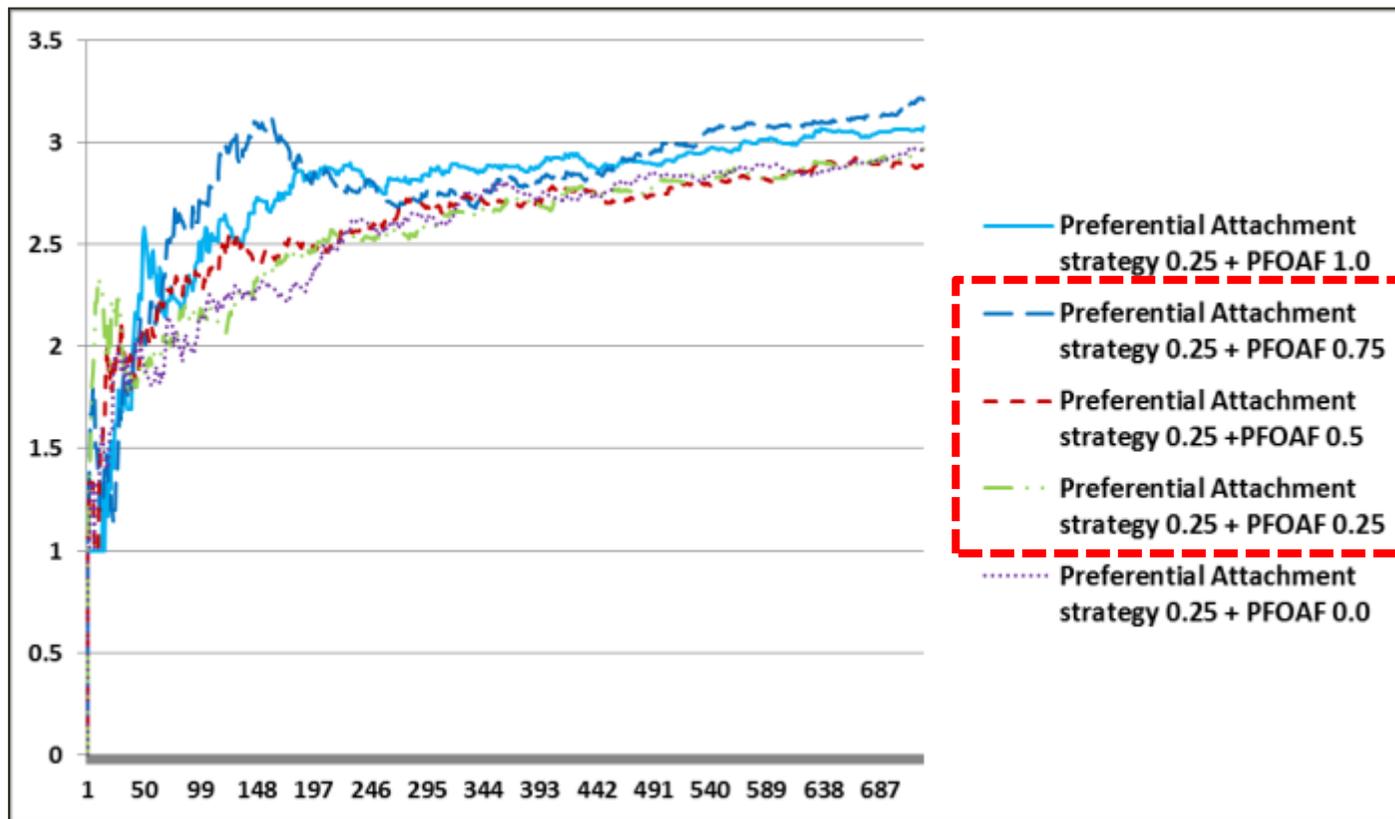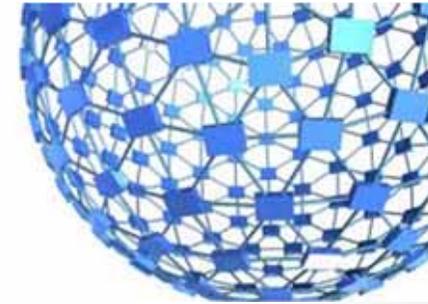
Model Parameters

- **Simulation Parameters**
    - We considered three parameters (i.e., P_{GM}, P_{FOAF} and P_{RAN}) in our network formation model,
        - The value of P_{GM} determines the rate of a network growth.
        - P_{FOAF} and P_{RAN} represent the rate of establishing potential links of FOAF and Random type.
        - The value of P_{RAN} is assumed to be equal to 1 - P_{FOAF}.
        - These parameters have values in the range [0, 1].

- **Structural Properties of Networks**
    - In this paper, we analyze one of the main structural properties of a network, called average shortest-path length (AVL). The shortest-path length (AVL) is defined as the shortest distance between node pairs in a network

$$AVL = \frac{1}{\frac{1}{2}N(N-1)} \sum_{i \geq j} l_{ij}$$

Result[1/4]

Legend:
- Preferential Attachment strategy 0.25 + PFOAF 1.0
- Preferential Attachment strategy 0.25 + PFOAF 0.75
- Preferential Attachment strategy 0.25 + PFOAF 0.5
- Preferential Attachment strategy 0.25 + PFOAF 0.25
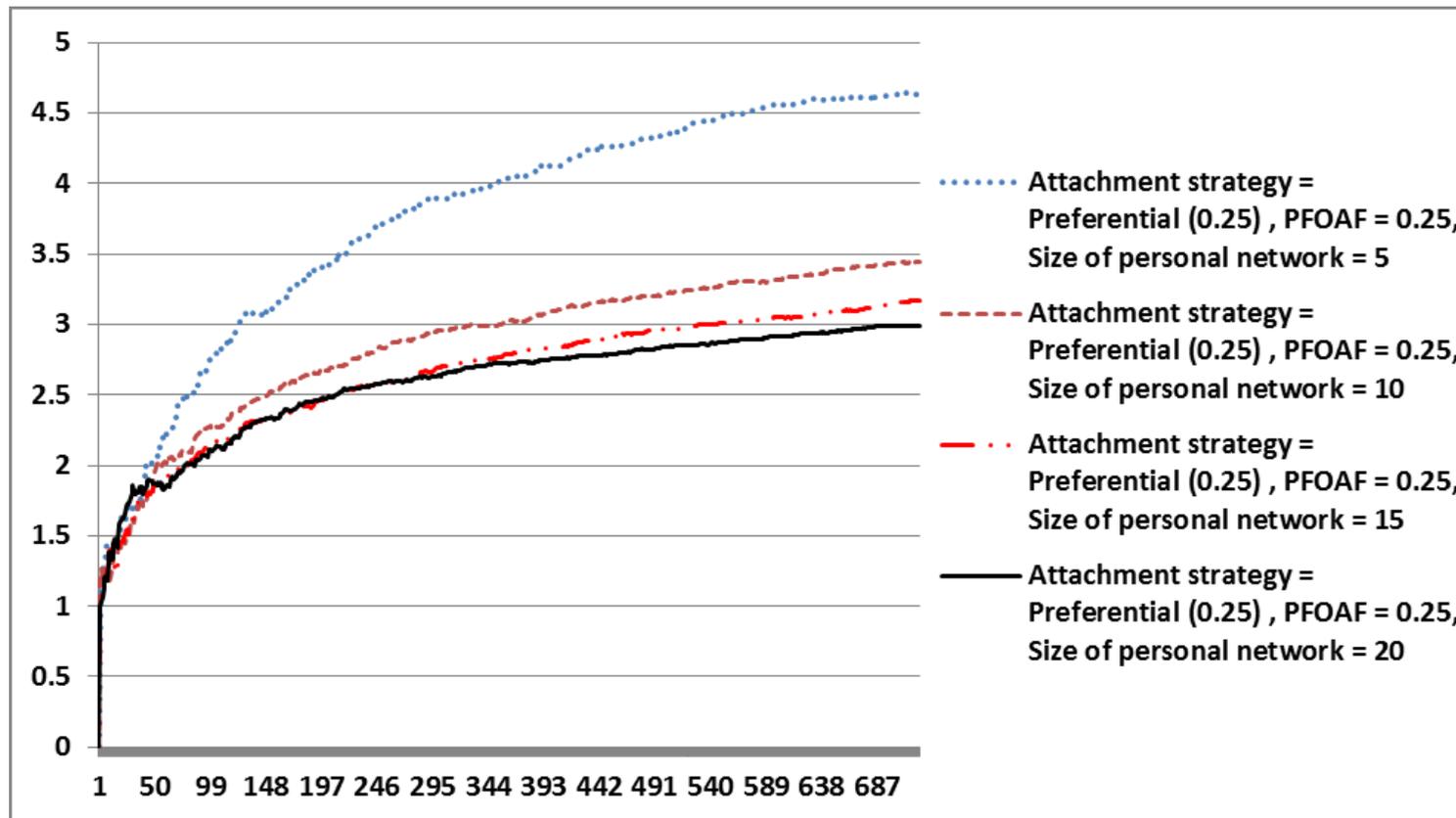- Preferential Attachment strategy 0.25 + PFOAF 0.0

Changes in AVL with respect to the preferential attachment growth model are shown. The x-axis depicts the simulation period, while the y-axis represents the AVL value.
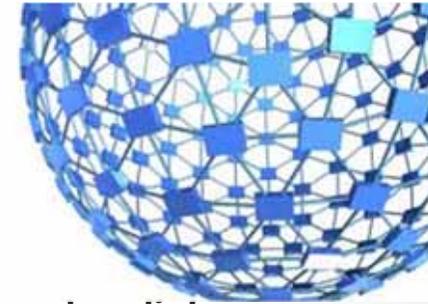
Result[2/4]
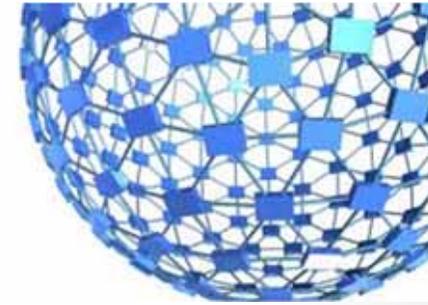
Limitations on the size of personal networks significantly changes the average path lengths among individuals.
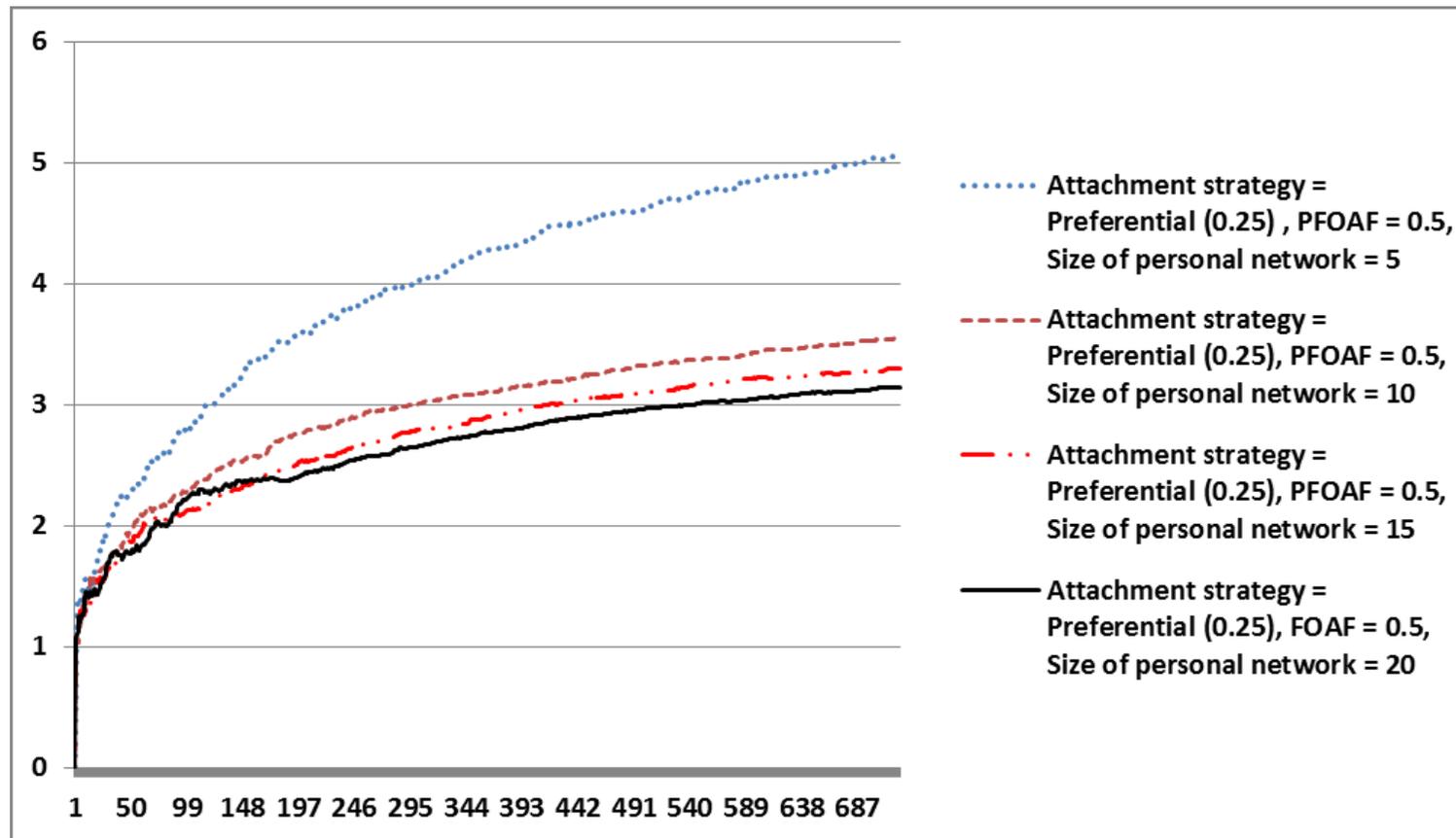

Changes in AVL with respect to the preferential attachment growth model for different sizes of the personal networks. The x-axis shows the simulation period, while the y-axis represents the AVL
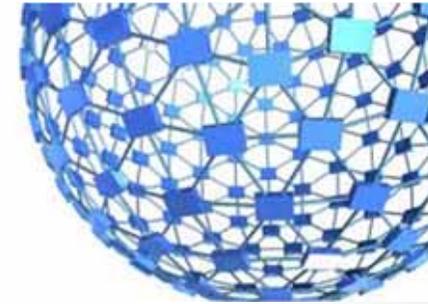
Result[3/4]

- **Similar trend As before, however, reduction in the percentage of random links increases the AVL.**

Changes in AVL with respect to the preferential attachment growth model for different sizes of the personal networks. The x-axis shows the simulation period, while the y-axis represents the AVL

Result[4/4]

- **Increases the AVL is move visible as a result of reduction in the percentage of random links**
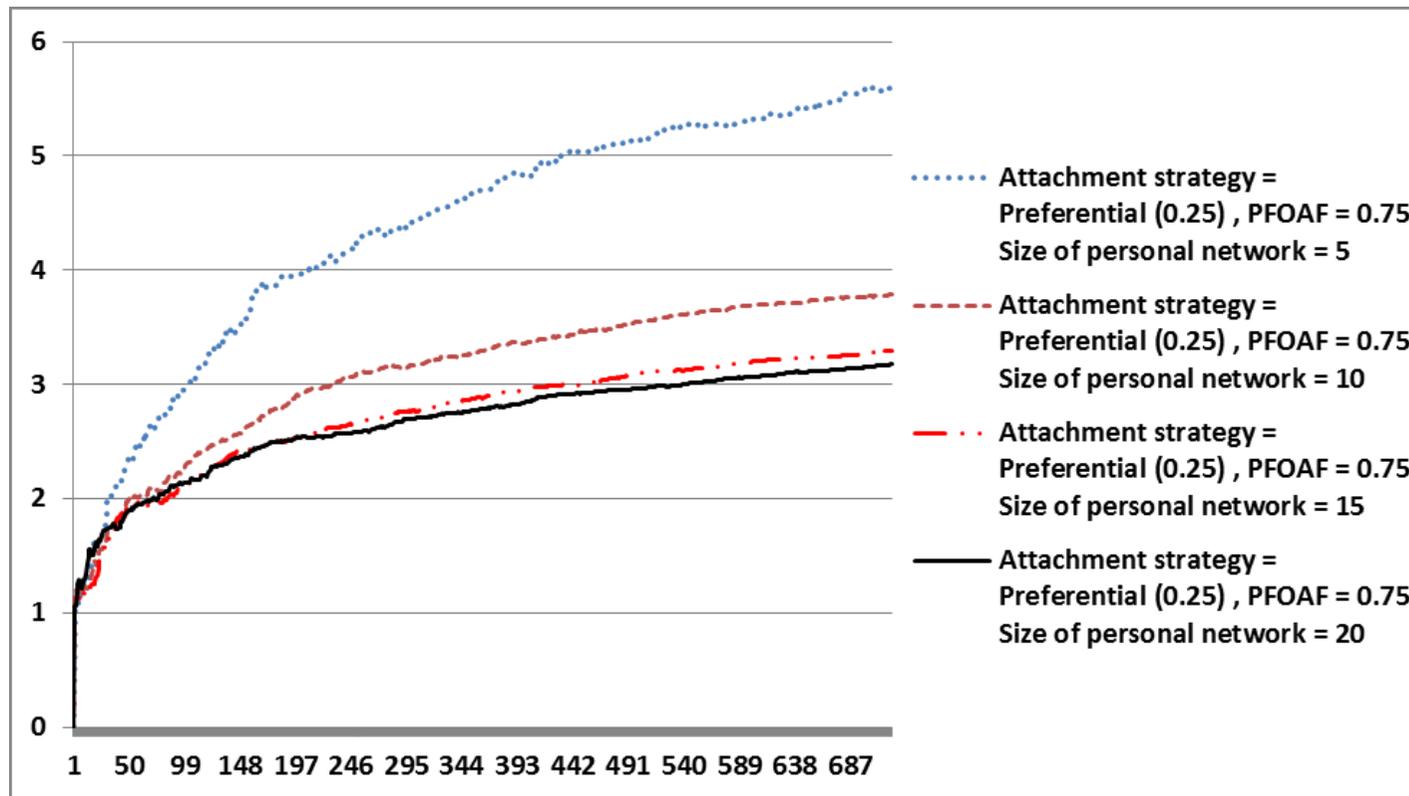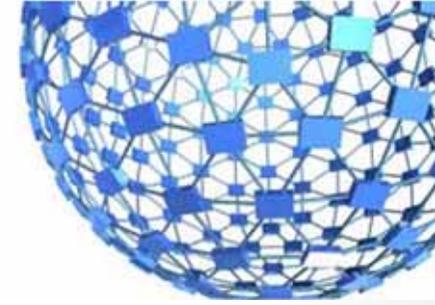

Changes in AVL with respect to the preferential attachment growth model for different sizes of the personal networks. The x-axis shows the simulation period, while the y-axis represents the AVL

Conclusion

- Although **we agree upon the fact that our world is getting smaller and smaller,** there is also a limit to how many close friends one can keep and count on.

- Besides, **having a high number of friends does not necessarily increase our trust circles.** In order to keep the boundary for our relationships, we usually determine features and patterns that distinguish a friend and a trusted contact.

- In addition to this, any personal network in the real world comes with the cost of maintaining the individual connections. **Maintenance of our personal network is not costless.** It requires spending time and effort.

15

Conclusion
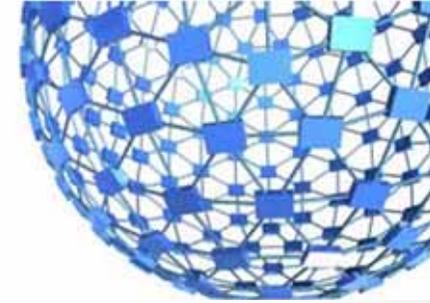
- We conducted numerical simulations to calculate and compare the average shortest-path length of individuals within the generated network.
  - The model was tested with forty configurations

- It was observed that,
  - **in addition to a different rate of variability in individual patterns of behavior in social environments,** **the limitations on the size of personal networks significantly changes the average path lengths among individuals**.

- As clearly observed,
  - **the network's average path length had a significantly smaller value, if the size of the personal networks has been set to a large value.** **Therefore, limitations on the size of personal networks lead to an increase in AVL value.**

Reference
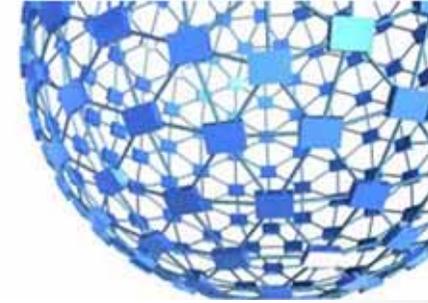
[1]Backstrom, L., Boldi, P., Rosa, M., Ugander, J., & Vigna, S. (2012, June). Four degrees of separation. In Proceedings of the 3rd Annual ACM Web Science Conference (pp. 33-42). ACM. DOI= http://dx.doi.org/10.1145/2380718.2380723

[2] Dunbar, R. I. M. 1992. Neocortex size as a constraint on group size in primates. Journal of Human Evolution 22 (6): 469–493. DOI= http://dx.doi.org/10.1016/0047-2484(92)90081-J

[3] Milgram, S. 1967. The small world problem. Psychology today, 2(1), 60-67. DOI= http://dx.doi.org/10.1037/e400002009-005

[4] Travers, J., and Milgram, S. 1969. An experimental study of the small world problem. Sociometry, 425-443. DOI= http://dx.doi.org/10.2307/2786545

[5] Kleinfeld, J. 2002. Could it be a big world after all? The six degrees of separation myth. Society, April, 12.

[6] Ugander, J., Karrer, B., Backstrom, L., and Marlow, C. 2011. The anatomy of the facebook social graph. arXiv preprint arXiv:1111.4503

[7] Dodds, P. S., Muhamad, R, and Watts, D. J. 2003. An exper-imental study of search in global social networks. Science, 301(5634), 827-829. DOI= http://dx.doi.org/10.1126/science.1081058

[8] Aristotle, Rackham, H., and Watt, S. 1996. The Nicomache-an Ethics. Ware, Hertfordshire: Wordsworth.

[9] Gilbert, N. 2006. Putting the Social into Social Simulation. Keynote address to the First World Social Simulation Con-ference, Kyoto.

[10] Watts, D. J., and Strogatz, S. H. 1998. Collective dynamics of 'small-world'networks. nature, 393(6684), 440-442. DOI= http://dx.doi.org/10.1038/30918

[11] Amaral, L. A. N., Scala, A., Barthélémy, M., and Stanley, H. E. 2000. Classes of small-world networks. Proceedings of the National Academy of Sciences, 97(21), 11149-11152. DOI= http://dx.doi.org/10.1073/pnas.200327197

[12] Barthélemy, M. 2003. Crossover from scale-free to spatial networks. EPL (Europhysics Letters), 63(6), 915. DOI= http://dx.doi.org/10.1209/epl/i2003-00600-6

[13] Wilensky U, NetLogo: center for connected learning and computer-based modeling. 1999. Northwestern University, Evanston, IL. http://ccl.northwestern.edu/netlogo/.

[14] Albert, R., and Barabási, A. L. 2002. Statistical mechanics of complex networks. Reviews of modern physics, 74(1), 47. DOI= http://dx.doi.org/10.1103/RevModPhys.74.47

[15] Barabási, A. L., and Albert, R. 1999. Emergence of scaling in random networks. Science, 286(5439), 509-512. DOI= http://dx.doi.org/10.1126/science.286.5439.509

[16] Barabási, A. L., Albert, R., and Jeong, H. 1999. Mean-field theory for scale-free random networks. Physica A: Statistical Mechanics and its Applications, 272(1), 173-187. DOI= http://dx.doi.org/10.1016/S0378-4371(99)00291-5

[17] Bishop, C. M., and Lasserre, J. 2007. Generative or discrimi-native? Getting the best of both worlds. Bayesian Statistics, 8, 3-23.